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A Bayesian Zero-Failure (BAZE) Reliability Demonstration Testing Procedure for Components of Nuclear Reactor Safety Systems

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NOTATION

a	- prior shape parameter
b	- prior scale parameter
b_0	- reference prior scale parameter
BAZE	- Bayesian Zero-Failure
BWR	- Boiling Water Reactor
CRBR	- Clinch River Breeder Reactor
ERDA	- Energy Research and Development Administration
f/h	- failures per hour
$f(\lambda)$	- prior probability density function of the failure-rate λ
$f(\lambda \mid 0 \text{ failures})$	- posterior probability density function of the failure-rate λ conditional on 0 observed failures in a test of nt unit-hours
γ	- posterior assurance
$(1.0-\gamma)$	- posterior risk
$I(a,x)$	- incomplete gamma function
	- $\int_0^x y^{a-1} e^{-y} dy$
k	- discrimination ratio (λ_1/λ_0)
λ	- failure-rate
λ_0	- specified failure-rate
λ_1	- criterion failure-rate $(k\lambda_0)$
λ^*, λ_*	- generic failure-rate

LL	- 50(1-p_0)th percentile of a gamma prior distribution; lower prior limit
LMFBR	- Liquid Metal Fast Breeder Reactor
MFR	- Median Failure Rate
MTTF	- Mean Time to Failure
n	- number of test units
n_0	- required number of test units
nt	- unit-hours of testing
$(nt)_0$	- required unit-hours of testing
p_0	- prior assurance
$P(\cdot)$	- probability
$P(x\ \text{failures}\mid\lambda)$	- conditional probability of x failures in a test of nt unit-hours with an underlying failure-rate of λ for each item
P(0 failures)	- unconditional probability of passing the BAZE test; unconditional probability of 0 failures in a test of nt unit-hours
$P(0\ \text{failures}\mid\lambda_*\leq\lambda\leq\lambda^*)$	- conditional probability of passing the BAZE test; conditional probability of 0 failures in a test of nt unit-hours
P(x failures)	- unconditional probability of x failures in a test of nt unit-hours
$P(\lambda_*\leq\lambda\leq\lambda^*\mid x\ \text{failures})$	- conditional probability that the failure-rate is contained in the interval $[\lambda_*,\lambda^*]$ given x failures in a test of nt unit-hours

POC	- Posterior Operating Characteristic
PWR	- Pressurized Water Reactor
θ_γ	- $100(\gamma)$th percentile of the standard gamma distribution
t_0	- required test duration
t	- test duration
UL	- $50(1+p_0)$th percentile of a gamma prior distribution; upper prior limit
X	- failure-time random variable

A BAYESIAN ZERO-FAILURE (BAZE) RELIABILITY DEMONSTRATION TESTING PROCEDURE FOR COMPONENTS OF NUCLEAR REACTOR SAFETY SYSTEMS

Abstract

A Bayesian-Zero-Failure (BAZE) reliability demonstration testing procedure is presented. The method is developed for an exponential failure-time model and a gamma prior distribution on the failure-rate. A simple graphical approach using percentiles is used to fit the prior distribution. The procedure is given in an easily applied step-by-step form which does not require the use of a computer for its implementation. The BAZE approach is used to obtain sample test plans for selected components of nuclear reactor safety systems.

I. INTRODUCTION

Most government and military contracts for hardware development include a numerical reliability requirement in the specifications. For example, a certain recent contract required an overall system failure-rate no larger than 11.64×10^{-6} f/h. In addition, most contracts require quantitative demonstrated assurance that such a requirement has been met. MIL-STD-781C[†] provides a standard which can be used to demonstrate such a requirement for times-to-failure that are exponentially distributed. The standard may be used for preproduction (qualification) tests, as well as production reliability acceptance (demonstration) tests. A typical reliability demonstration statement is that a failure-rate requirement of X failures/h be demonstrated with Y% confidence.

The purpose of this report is to develop a Bayesian reliability demonstration testing procedure for exponentially distributed failure times which can be easily and effectively used to demonstrate component/subsystem/system reliability conformance to stated requirements. The procedure will also be used to develop suggested test plans for various components used in nuclear power reactor safety systems. This procedure may be used to verify specified and projected component failure-rates in LMFBR safety systems. However, the procedure is a general one and its use is not restricted to nuclear power

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[†]MIL-STD-781C "Reliability Qualification and Production Acceptance Tests: Exponential Distribution," Washington, D.C.: U.S. Government Printing Office, (Draft), August, 1976.

safety systems. It may also be used to demonstrate reliability for such equipment categories as ground equipment, shipboard equipment, avionic equipment, weapons systems, and surveillance equipment.

Over the past two decades numerous classical reliability demonstration testing methods have been devised for various failure-time distributions. Classical demonstration test plans for components having an exponential failure-time distribution (constant failure-rate) may be fixed time (Type I Censoring), fixed number of failures (Type II Censoring), or sequential tests. In addition, such tests may be conducted either with or without the replacement of failed items when failures occur during the test. One example of a classical procedure is MIL-STD-781C which gives various test procedures derived under the assumption of a constant failure-rate. These tests are based on various levels of producer's and consumer's risk, design ratios, and the confidence level of the test. In these tests, a simple statistical hypothesis of the form

$$H: \lambda=\lambda_0 \text{ (specified failure-rate)}$$
$$A: \lambda=\lambda_1 \text{ (maximum acceptable failure-rate)}$$

is tested, where λ is the (unknown) failure-rate of the device and $\lambda_1>\lambda_0$. The producer's risk is the probability that, if H is true, A will be accepted, while the consumer's risk is the probability that, if A is true, H will be accepted. The design ratio is defined as the ratio of λ_1 to λ_0. The text by Mann, Schafer, and Singpurwalla (1974) gives an excellent discussion of the basis upon which these and other classical test plans have been developed.

In practice, it is often the case that the reliability parameters of interest, such as MTTF, reliable life, failure-rate, etc., most realistically should be treated as random variables and not as constant values. The statistical distribution which expresses the true underlying variation in the parameter is called the prior distribution of the parameter (when treated as a random variable). This approach has been taken in previous reactor safety analyses, such as the Rasmussen study.* In that study, the prior distribution for the reactor component failure-rates was taken to be the log-normal distribution.** Such an approach permits the reliability quantity of interest to vary randomly due to such factors as environmental effects, plant-to-plant differences, maintenance effects, and different operational demands.

By treating the parameter as a random variable, a Bayesian approach may be considered. The main advantage of the Bayesian approach is that the resulting estimates are computed from all available information, and not just narrowly defined test data

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**Ibid., p. II-40

of precise content. Rather, there exist two sources of information regarding the Bayesian procedure. One source of information, the so-called "prior information," expresses the sum total of engineering judgment and belief concerning the underlying prior distribution of the parameter of interest. It is precisely this distribution which expresses the inherent variability of the parameter itself. The other source of information is the statistical model used to describe either the time-to-failure data or the test results themselves. Both sources of information are combined via Bayes Theorem [see Waller and Martz (1975)] to produce a statement such that the probability that the failure-rate does not exceed the specified value is Y%.

Such an approach is particularly applicable for deriving test plans for demonstrating the component failure rates of proposed nuclear reactors. The reason for this is that failure data are becoming available for similar components in use in existing power reactor systems throughout the world. These data, which are continuously being compiled and reported in numerous data base systems, represent the "prior information" for similar components to be used in advanced reactor systems, such as LMFBR systems.

The resulting Bayesian test plans are generally resource-effective, due to the use of all available information and judgment concerning the parameter of interest. If the prior information supports an adequately reliable component, then less testing will usually be required compared to the classical case.

If the opposite is true, then more testing may be required. A general introduction to the use of Bayesian methods in reliability is given by Waller and Martz (1975), (1976a), (1976b).

A brief review of Bayesian reliability demonstration procedures is given in Section II. A procedure for choosing the prior distribution is presented in Section III. The BAZE procedure is developed in Section IV. Section V contains an example illustration of the method, as well as an examination of the sensitivity of the results to the chosen prior distribution. Appropriate prior distributions are fitted to various components used in nuclear power systems in Section VI. Section VII presents a selection of sample BAZE test plans for these reactor components.

II. BAYESIAN RELIABILITY DEMONSTRATION TESTING

The state of the art of Bayesian reliability demonstration test procedures will now be reviewed. One of the earliest references to Bayesian reliability demonstration plans is that of Bonis (1966). Since that time numerous Bayesian schemes have been developed. Easterling (1970) presented a somewhat modified Bayesian demonstration procedure. Schafer and Singpurwalla (1970) developed a sequential Bayes procedure for obtaining required test plans. Schafer (1969), (1971), and (1973) has considered three types of Bayesian plans: (1) Bayesian fixed time tests, (2) mixed Bayesian/classical, and (3) Bayesian sequential tests. Following along these same lines, Goel *et al.*, (dates unknown) developed Bayesian plans for slightly different criteria. Blumenthal (1973) has also developed Bayesian test plan procedures. Guild (1968), (1973) has developed what he refers to as "median failure rate" (MFR) reliability demonstration plans. Other Bayesian plans have also been considered by Balaban (1969), (1975) and Ramos (1970). Joglekar (1975) discusses several of these Bayesian testing schemes. Recently, Goel and Joglekar [1976] have prepared a comprehensive account of the state of the art of Bayesian reliability acceptance sampling. This five-part series provides an excellent introduction to the subject.

One of the major problems with most Bayesian test plans is the relative difficulty in obtaining a desired plan in practice. This is due to the presence of additional prior parameters, as well as the relative complexity of the method. For example, most of the above Bayesian plans are derived for a constant failure-rate model and a gamma prior distribution on the failure-rate [see Waller and Martz (1975)].

The gamma prior distribution is the natural conjugate prior distribution for the constant failure-rate model. Schafer (1969) investigated data from 32 different equipments and found that in 29 cases a gamma prior distribution adequately fit the data. Others have likewise observed the suitability and versatility of the gamma prior distribution. For these reasons it is also considered here. However, it is frequently not an easy task to identify an appropriate gamma prior distribution and, once this has been done, to obtain the required test plan. The choice of test plan criteria, e.g., the consumer and producer risks that are to be controlled, can also complicate the determination of the required plan. Certain criteria yield plans that are simple to determine. One of the easiest of the above Bayesian procedures to use in practice is that given by Guild (1973). A more general version of this procedure is developed in Section IV for use here. An earlier version of this BAZE procedure was presented by Martz and Waller (1976c). The current BAZE procedure contains a more useful and practical procedure for selecting the prior distribution, which has been incorporated into the BAZE procedure itself. The new procedure also contains a simple method for examining the sensitivity of the resultant BAZE test plan to the chosen prior distribution. This serves to make the method more useful in practice.

Three somewhat distinctive aspects of the procedure should be mentioned before we begin the development. First, the criterion upon which the procedure is based is simple, pertinent, and easy to grasp. Second, the fitting of the prior distribution is an integral part of the procedure and is based on the use of information regarding two percentiles of this distribution. Third, the procedure is straightforward and easy to apply in practice, with only a few simple graphs and tables and pocket calculator required. Together these provide a useful Bayesian procedure for a large variety of applications.

III. SELECTING A GAMMA PRIOR DISTRIBUTION

We assume that the time-to-failure of interest is an exponentially distributed random variable with failure-rate parameter λ. For Bayesian analyses in this model, the family of gamma distributions with probability density functions given by

$$f(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, \; \lambda, \; a, \; b > 0 \tag{1}$$

provides conjugate prior models for λ. In practice, an engineer must select a member of this family as the prior distribution to be used in determining the BAZE test plan to be discussed in the next section. The selection of a particular prior distribution is accomplished by identifying values for the prior shape parameter a and prior scale parameter b. The parameter a can be further interpreted as the number of pseudo failures in a prior life test of duration b pseudo hours. The mean and variance of λ are given by (a/b) and (a/b^2), respectively.

Some additional benefits in using a gamma prior distribution are as follows:

a. The two parameters give sufficient flexibility to model a variety of shapes of prior distributions likely to be encountered in practice. The following indicate the possible shape characteristics of a conjugate gamma prior distribution.

	0<a<1	a=1	a>1
b>0	L-shaped or decreasing	Exponential	Unimodal with mode at b(a-1)

b. The positive skewness can account for general behaviors of assessed data in which less likely but large deviations may occur (such as abnormally high failure-rates due to batch defects, environmental degradation, and other outlier causing effects).

c. In practice, the gamma family often satisfactorily fits observed data [see Schafer (1969)].

d. The positively skewed nature of the gamma family provides a protective, positive-type bias which is retained when the distribution is propagated by means of a Bayesian analysis.

A simple method for determining values for a and b will now be described. The method requires an engineer to provide upper and lower percentile values. Once these are given, a simple graphical or table look up, in addition to a few simple calculations, yields the corresponding values of a and b.

The engineer must provide two values of the failure-rate λ, referred to as the <u>lower prior limit</u> (LL) and <u>upper prior limit</u> (UL), such that

$$P(\lambda < LL) = P(\lambda > UL) = (1.0-p_0)/2, \tag{2}$$

where p_0 is required to be equal to one of the values 0.95, 0.90, or 0.80, and where $LL < UL$. That is, LL and UL are specified such that there is an equal $50(1.0-p_0)$% chance that the true (unknown) failure-rate is either less than LL or greater than UL, respectively. Thus LL and UL are the respective $50(1.0-p_0)$th and $50(1.0+p_0)$th percentiles of the prior gamma distribution. For example, suppose that an engineer's best prior judgement or belief is that $P(\lambda < 1.0 \times 10^{-7} f/h) = 5\%$ and that $P(\lambda > 1.0 \times 10^{-5} f/h) = 5\%$. Thus $(1.0-p_0)/2 = 0.05$, $p_0 = 0.90$, $LL = 1.0 \times 10^{-7}$ f/h, and $UL = 1.0 \times 10^{-5}$ f/h. The quantity $100p_0$% is the prior assurance that the interval (LL, UL) contains the failure-rate of interest. Since engineers are increasingly becoming accustomed to working with 5% error probabilities, it is likely that $p_0 = 0.90$ will normally be used. However, 80% and 95% prior assurances can also be used. For example, in the Rasmussan study (WASH-1400), 90% prior assurance was considered. If the prior assurance is free to be selected, it is recommended that $p_0 = 0.90$ be used. In this case, LL and UL become the lower and upper prior 5% bounds, respectively, on the failure-rate.

A mathematical justification of the procedure to be described is given in Appendix B. A step-by-step outline of the procedure is as follows:

Step 1: Specify the values of LL, UL, and $p_0 = 0.80$, 0.90, or 0.95 that represent the totality of your best judgement and belief about the failure-rate λ of interest. These values are selected in accordance with (2).

Step 2: Compute the value of $\log_{10}$ (UL/LL).

Step 3: For the value of p_0 chosen in Step 1 and the value of $\log_{10}$ (UL/LL) calculated in Step 2, read the required value of shape parameter a from Figure A1 (see Appendix A).

Step 4: For the value of p_0 from Step 1 and for the value of a found in Step 3, read the value of b_0 from Figure A2 (see Appendix A).

Note: Table A2 (see Appendix A) may be used in lieu of Figure A2 to obtain b_0, depending upon which is more convenient to use. If necessary, interpolate in Table A2.

Step 5: For the value of LL from Step 1 and the value of b_0 from Step 4, calculate the required value of the scale parameter b according to

$$b = b_0(1.0\text{x}10^{-6}\text{f/h})/\text{LL}.$$

Let us illustrate this procedure by means of an example.

Example: For a certain component of interest, suppose it is is believed that the failure-rate λ is such that $P(\lambda < 1.0x10^{-7} f/h) = 5\%$ and $P(\lambda > 1.0x10^{-5} f/h) = 5\%$. It is required to identify the particular gamma distribution which is consistent with this belief.

Step 1: $LL = 1.0x10^{-7} f/h$, $UL = 1.0x10^{-5} f/h$, and $p_0 = 0.90$.

Step 2: $\log_{10}(UL/LL) = \log_{10}(10^2) = 2.0$.

Step 3: From Figure A1, for $p_0 = 0.90$ and $\log_{10}(UL/LL) = 2.0$, we find $a = 0.84$.

Step 4: For $a = 0.84$ and $p_0 = 0.90$, Table A2 yields $b_0 = 2.6723x10^4 h$.

Step 5: The required scale parameter b becomes

$$b = \frac{(2.6723x10^4 h)(1.0x10^{-6} f/h)}{(1.0x10^{-7} f/h)}$$

$$= 2.6723x10^5 \text{ h}.$$

By means of the incomplete gamma function code INCGAM (written by D.E. Amos and S.L. Daniel of Sandia Laboratories, Albuquerque, NM, November 1974), the actual tail-area probabilities for a gamma distribution with parameters $a = 0.84$, $b = 2.6723x10^5$ h are 0.05, as desired. However, this is not always the case. Due to numerical and round-off errors, the upper tail area may not be exactly equal to $(1 - p_0)/2$. In Step 5, the denominator of the expression for b was LL. This was done to insure that the lower tail area will always be $(1-p_0)/2$, while the upper tail area may depart somewhat from the desired value $(1-p_0)/2$. We chose to hold the lower tail area fixed because of the positively skewed nature of the gamma distribution.

One final note concerns the usefulness of Figures A1 and A2 in practice. The effective range of values of a considered in Figures A1 and A2 is between 0.25 and 10.0. Experience with fitting gamma prior distributions to failure-rate data indicates that this range should contain nearly all situations likely to be encountered in practice. This range is consonant with ratios of UL to LL roughly between 0.3 and 4.0 orders of magnitude (powers of 10).

IV. BAYESIAN ZERO-FAILURE (BAZE) RELIABILITY DEMONSTRATION TESTING

The BAZE reliability demonstration procedure was developed by Martz and Waller (1976c). This procedure is given here in an expanded form which includes the procedure for fitting a gamma prior distribution, described in the preceding section, and other features as well.

This section describes how to construct and apply Bayesian fixed time demonstration test plans of the replacement type, called BAZE plans, for systems/subsystems/components having a constant failure-rate. The BAZE procedure is appropriate for testing time-dependent chance failure mechanisms.

To begin, consider a device, henceforth referred to as a "component," having an exponential failure time distribution with failure-rate λ. Thus, the failure-time random variable X of this component is assumed to follow the well-known exponential probability density function given by

$$f(x|\lambda) = \lambda e^{-\lambda x}, \; x \geq 0, \; \lambda > 0. \tag{3}$$

As mentioned earlier, it is assumed here that the prior distribution of λ is the natural conjugate gamma distribution with probability density function given by

$$f(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, \; \lambda, \; a, \; b > 0, \tag{4}$$

where a is the prior shape parameter and b is the prior scale parameter.

The test plans considered here assume that n identical components are tested each for a prespecified length of time t, the test duration. The quantities n and t are to be determined consistent with the following statistical test criterion. The test criterion is as follows: if no failures occur, the test is passed, while if one or more failures occur, the test is failed. Thus the test is terminated either at the prespecified test time t or at the time of the first component failure, whichever occurs first. Such "zero-failure" test plans usually require the smallest unit-hour test combination nt for a stated confidence, and are thus test resource-effective. In addition, by restricting consideration to zero-failures, such test plans are easy to obtain. Now the probability of obtaining exactly zero failures during the test is given by

$$P(0 \text{ failures} | \lambda) = e^{-nt\lambda}. \tag{5}$$

The posterior distribution of λ is also a gamma distribution with scale parameter $(b + nt)$ and shape parameter a. Thus, conditional on zero failures in nt unit-hours of testing, the posterior probability density function of λ becomes

$$f(\lambda | 0 \text{ failures}) = \frac{(b+nt)^a}{\Gamma(a)} \lambda^{a-1} e^{-(b+nt)\lambda}. \tag{6}$$

In order to find the required unit-hour test combination nt, some criterion regarding the desired confidence level of the demonstration test must be given. The plans presented here satisfy the <u>posterior risk criterion</u> given by

$$P(\lambda \leq k\lambda_0 | 0 \text{ failures}) = \gamma, \tag{7}$$

where $P(\cdot)$ is a probability function. Here λ_0 is the <u>specified failure-rate</u>. If we define $\lambda_1 \equiv k\lambda_0$, then λ_1 is referred to as the test <u>criterion failure-rate</u> and k is known as the <u>discrimination ratio</u>. The test criterion in (7) is interpreted as follows. In a test that is passed, i.e., zero failures occur, the probability is 100γ% that the component failure-rate does not exceed $(k\lambda_0)$. Here $(1.0-\gamma)$ will be referred to as the <u>posterior risk</u> and γ will be referred to as the posterior assurance. The word "posterior" denotes that the assurance pertains to tests which have been passed. Of course, values of γ and k are required in order to determine the required test plan. More will be said about the selection of these values later.

Recall that the test procedure requires that n items be placed on life test for t hours. If no failures occur, the test is passed. Now, if the test is passed, it may be claimed that a failure-rate not exceeding $(k\lambda_0)$ has been demonstrated with 100γ% posterior assurance. If a single failure occurs, the test is failed, and the forgoing claim cannot be made.

The BAZE procedure described requires specification of values for the following five quantities:

- LL(lower prior limit)
- UL(upper prior limit)
- p_0(the prior assurance)
- λ_1(the criterion failure-rate)
- γ(the posterior assurance).

It is noted that the criterion failure-rate λ_1 may be equal to the specified failure-rate λ_0. In this case k=1; otherwise, $k \neq 1$. The procedure is developed by writing

$$p(\lambda \leq \lambda^* | 0 \text{ failures}) = \int_0^{\lambda^*} \frac{(b+nt)^a}{\Gamma(a)} \lambda^{a-1} e^{-(b+nt)\lambda} d\lambda$$

$$= \frac{I(a, [b+nt]\lambda^*)}{\Gamma(a)}, \qquad (8)$$

where I(a,x) is the widely studied incomplete gamma function defined by

$$I(a, x) = \int_0^x y^{a-1} e^{-y} dy. \quad (9)$$

Tables and computer routines for evaluating this function are widely available for use in our development. Hence, when LL, UL, p_0, γ, and λ_1 are specified, the step-by-step procedure for obtaining the required BAZE test plan is as follows:

Step 1: For the specified values of LL and UL, compute the value of $\log_{10}$(UL/LL).

Step 2: For p_0 = 0.80, 0.90, or 0.95, and the value of $\log_{10}$(UL/LL) from Step 1, obtain the value of the prior shape parameter a from Figure A1 (Appendix A).

Step 3: For p_0 and the value of a from Step 2, obtain the value of b_0 from either Figure A2 or Table A2.

Step 4: For the value of LL and b_0 from Step 3, calculate the value of the prior scale parameter b, in appropriate time units, according to

$$b = b_0 \ (1.0 \times 10^{-6} f/h)/LL.$$

Step 5: Obtain the value of θ_γ from Table A1 (Appendix A) for the value of a found in Step 2 and γ. <u>Note:</u> Table A1 may be used directly for a = 0.0001

(0.0001) 0.01 (0.001) 0.10 (0.01) 1.0 (0.1) 5.0 (0.5) 10.0 (1.0) 50.0 and $\gamma = 0.99$, .975, .95, .90, .85, .80, .75, .70, .60, .50. For other values of a and/or γ, either interpolate in Table A1 or solve the equation given by

$$\int_0^{\theta_\gamma} \lambda^{a-1} e^{-\lambda} d\lambda - \gamma\Gamma(a) = 0$$

for θ_γ. It is mentioned here that in constructing Table A1 the incomplete gamma function in the above equation was numerically calculated by use of the code INCGAM, written by D.E. Amos and S.L. Daniel of Sandia Laboratories, Albuquerque, NM, November 1974. The above equation was solved on a CDC 6600 computer by use of the root-solving code ZEROIN, written by L.F. Shampine and H.A. Watts, also of Sandia Laboratories, September 1970.

Step 6: With λ_1 and the θ_γ value from Step 5, and the value of b from Step 4, solve for the required BAZE unit-hours of test $(nt)_0$ given by

$$(nt)_0 = (\theta_\gamma - b\lambda_1)/\lambda_1.$$

Note: Negative values of $(nt)_0$ can occur. A negative value of $(nt)_0$ can be interpreted as a demonstration of the failure-rate λ_1, at the stated posterior assurance level, without the need for further testing. This situation occurs whenever the prior distribution satisfactorily meets the assurance requirement.

Step 7: Calculate the sensitivity of the final plan and the posterior assurance to errors in the prior assurance, as well as the sensitivity of the final plan to errors in the posterior assurance and criterion failure-rate (cf Section V. Example).

Step 8: Identify the final plan to be used based on the results of the sensitivity analysis conducted in Step 7 and the unconditional probability of passing the test [from (10) or (11) below].

Step 9: The required test duration t_0 and number of test units n_0 is given by any pair of values satisfying $n_0 t_0 = (nt)_0$, where $(nt)_0$ is the required unit-hours of test from Step 8 and n_0 is a positive integer. The values n_0 and t_0 are selected by outside considerations, such as test time constraints and the number of test units available.

It is noted here that if $a = 1$, $b = 0$, and $\gamma = 0.50$, then the BAZE test plan is exactly the same as the classical test plan. If $a = 1$ and $\gamma = 0.50$, then the BAZE test plan will always require less unit-hours of testing, depending upon the magnitude of b.

For a specified fixed value of a, the required BAZE unit-hours of test decreases as γ decreases, b increases, or k increases. Consequently, for given values of a and b, an opportunity is present for the test designer to trade between decreasing testing costs [decreasing $(nt)_0$] and decreasing test assurance [decreasing γ and/or increasing λ_1]. Such tradeoffs are illustrated in Section V.

Suppose that high-reliability components with failure-rate on the order of magnitude of 10^{-6} f/h are being considered. If the prior mean is in this range, then it is true that as the spread between LL and UL increases both a and b will generally decrease. This is seen in Figures A1 and A2. In fact, in situations such as this, a will frequently be less than one. This situation occurs whenever the prior variance is quite large, i.e., whenever the prior distribution is diffuse [see Waller and Martz (1975)]. Such situations frequently occur in reliability and this fact has motivated the fine grid of a values less than one considered in Table A1.

A quantity of particular interest to the producer is the unconditional probability of passing the test when using a test plan with (nt) unit-hours of test. The unconditional probability of <u>not</u> passing the test is, in some sense, the "producer's risk" of the BAZE procedure. The probability of passing the test must be sufficiently large in order that the producer be willing to conduct the test. This probability also conveys to the consumer the likelihood that the required posterior assurance will be realized. This probability is given by

$$P(\text{Passing the Test}) \equiv P(0 \text{ failures}) = [b/(b+nt)]^a. \quad (10)$$

Related to this quantity is the conditional probability of passing the test when it is known that the true (unknown) failure-rate λ lies within a given interval $[\lambda_*, \lambda^*]$, where $0 \leq \lambda_* < \lambda^* \leq \infty$. In this case we have

$$P(\text{Passing the Test} | \lambda_* \leq \lambda \leq \lambda^*) = \left(\frac{b}{b+nt}\right)^a \left[\frac{I(a,[nt+b]\lambda^*) - I(a,[nt+b]\lambda_*)}{I(a,b\lambda^*) - I(a,b\lambda_*)}\right], \quad (11)$$

where $I(a,x)$ is defined in (9). It is also noted that, if $\lambda_* = 0$ and $\lambda^* = \infty$, then the conditional probability of passing the test given in (11) reduces to the unconditional probability given in (10). In practice, an interval $[\lambda_*, \lambda^*]$ which is certain to contain the failure-rate can frequently be identified. If this can be done, then (11) should be used in place of (10).

What posterior assurance do we have about the failure-rate if one or more failures occur during the test, i.e., if the test is failed? Suppose that failed items are replaced as they occur during the test. Then

$$P(x \text{ failures} | \lambda) = \frac{e^{-nt\lambda}(nt\lambda)^x}{x!}, \quad x = 0, 1, \ldots, \quad (12)$$

and the posterior probability density function of λ becomes

$$f(\lambda | x \text{ failures}) = \frac{(b+nt)^{a+x}}{\Gamma(a+x)} \lambda^{a+x-1} e^{-(b+nt)\lambda}. \quad (13)$$

Now, for any specified interval $[\lambda_*, \lambda^*]$, where $0 \leq \lambda_* < \lambda^* \leq \infty$, we have

$$P(\lambda_* \leq \lambda \leq \lambda^* | x \text{ failures}) = \tag{14}$$
$$\frac{1}{\Gamma(a+x)} \left[I(a + x, [b + nt]\lambda^*) - I(a + x, [b + nt]\lambda_*) \right].$$

In particular, we have

$$P(\lambda \leq k\lambda_0 | x \text{ failures}) = I(a + x, [b + nt]k\lambda_0)/\Gamma(a + x). \tag{15}$$

It is observed that, if $x = 0$, then (15) is equal to the specified posterior assurance γ. For the criterion failure-rate $(k\lambda_0)$, as x increases (15) becomes smaller than γ. Thus, as more failures occur, we have less posterior assurance about the failure-rate not exceeding the criterion value.

Also, the unconditional probability of obtaining exactly x failures in a test of nt unit-hours duration is

$$P(x \text{ failures}) = \frac{b^a (nt)^x \Gamma(a + x)}{\Gamma(a)\Gamma(x + 1)(nt + b)^{a + x}}. \tag{16}$$

The statistical performance characteristics of the chosen plan are completely summarized by means of the <u>posterior operating characteristic</u> (POC) curve. This curve is obtained by plotting $P(\lambda \leq \lambda^* | 0 \text{ failures})$ as a function of λ^*. Unlike classical OC curves, the POC curve is a cumulative distribution function. This probability may be computed from (8).

We cannot emphasize enough that both the consumer and producer must be willing to pay the price for increasing assurance of small failure-rates by increasing the unit-hours of testing. This will be illustrated in the example in the next section.

V. EXAMPLE

In order to fully illustrate the BAZE procedure, consider the following example. Consider a certain component whose random failure-rate is required to be demonstrated. How many unit-hours of testing with no countable failures are required in order to be able to claim that $P(\lambda \leq 2.0 \times 10^{-6} f/h) \geq 0.70$, after the test has been passed? From past experience and engineering judgement, suppose it is believed that

$$P\{\lambda \leq 8.5 \times 10^{-8} f/h\} = P\{\lambda \geq 4.8 \times 10^{-6} f/h\} = 5\%.$$

Thus $\lambda_1 = \lambda_0 = 2.0 \times 10^{-6} f/h$, $\gamma = 0.70$, $LL = 8.5 \times 10^{-8} f/h$, $UL = 4.8 \times 10^{-6} f/h$, and $p_0 = 0.90$. Following the step-by-step procedure in the preceding section yields the following results:

Step 1: $\log_{10}(UL/LL) = 1.75$.

Step 2: For $\log_{10}(UL/LL) = 1.75$ and $p_0 = 0.90$, we read $a = 1.0$ from Figure A1.

Step 3: For $p_0 = 0.90$ and $a = 1.0$, we obtain $b_0 = 5.1293 \times 10^4 h$ from Table A2.

Step 4: For $LL = 8.5 \times 10^{-8} f/h$ and $b_0 = 5.1293 \times 10^4 h$, we calculate

$$b = (5.1293 \times 10^4 h)(1.0 \times 10^{-6} f/h)/(8.5 \times 10^{-8} f/h)$$

$$= 603.44 \times 10^3 h.$$

Step 5: For $a = 1.0$ and $\gamma = 0.70$, we obtain $\theta_{0.70} = 1.203973$ from Table A1.

Step 6: For $\lambda_1 = 2.0x10^{-6}f/h$, $\theta_{0.70} = 1.203973$, and $b = 0.60x10^6h$, the required BAZE unit-hours of test $(nt)_0$ are calculated to be

$$(nt)_0 = [1.203973 - (0.60x10^6h)(2.0x10^{-6}f/h)]/(2.0x10^{-6}f/h)$$

$$= 1987 \text{ unit-hours.}$$

The POC curve for this plan is plotted in Figure 1.

Step 7: Let us first examine the sensitivity of the BAZE plan to changes in the posterior assurance γ and criterion failure-rate λ_1. We express the varying criterion failure-rate λ_1 as a function of k according to $\lambda_1 = k\lambda_0 = k(2.0x10^{-6}f/h)$. Table I gives the resultant test plan as a function of a selected grid of values of k and γ. The plan $(nt)_0$ = 1987 unit-hours is indicated in table for k = 1.0 and $\gamma = 0.70$. It is clearly observed that the required unit-hours of test increase as γ increases and k decreases. It is observed that the optimal test plan is quite sensitive to increasing γ and also somewhat sensitive to decreasing k, for the range of k indicated. The "zeros" indicate those situations in which the prior distribution is sufficient to guarantee that the risk is at or below the specified level, without the need for additional testing.

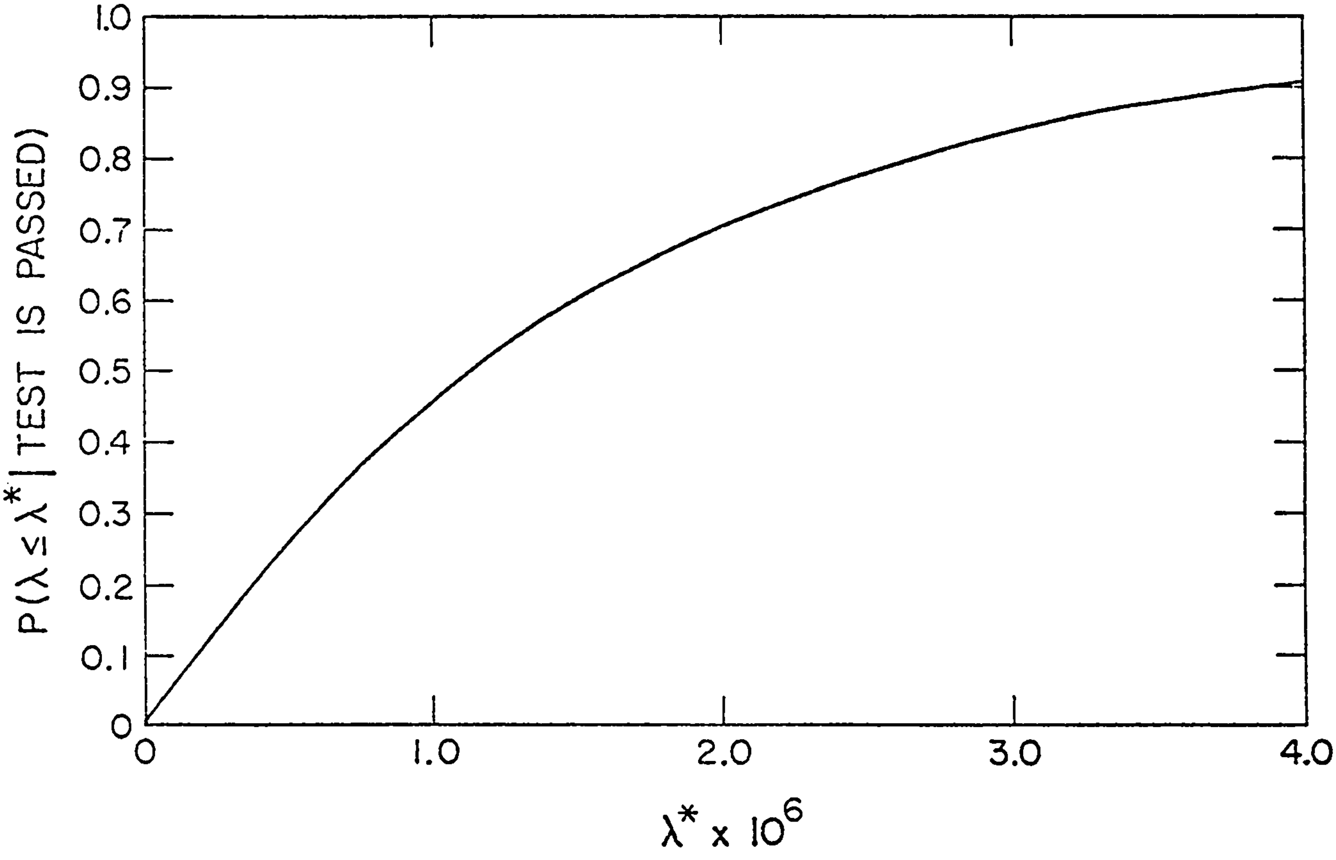

Fig. 1. The POC curve for Test Plan $(nt)_o$ = 1987 Unit-Hours of Test.

TABLE I

REQUIRED UNIT-HOURS OF TESTING FOR
SELECTED VALUES OF K AND γ
($a = 1.0, b = 0.60 \times 10^6$)

γ	k										
	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	4005170	3237642	2699407	2278231	1953428	1702595	1493259	1318821	1171219	1044704	935057
.975	3038879	2474056	2034914	1705549	1449377	1244440	1076763	937033	813300	717457	524526
.950	2395732	1896443	1539009	1272333	1064246	897866	761696	648222	552205	459906	390577
.900	1772565	1318921	1044704	839116	679214	551293	446630	359410	283010	222352	167528
.850	1297120	980933	755086	595700	453936	345560	262327	190467	124662	77543	32373
.800	1079439	741198	549599	405899	294132	204719	131563	70599	19015	0	0
.750	786294	555245	390210	266434	170163	93147	30134	0	0	0	0
.700	503973	403311	259981	152483	68874	1987	0	0	0	0	0
.600	316291	163576	54494	0	0	0	0	0	0	0	0
.500	93147	0	0	0	0	0	0	0	0	0	0

Let us now determine the sensitivity of the BAZE plan to changes in the prior assurance. Suppose that, regarding the prior distribution, each tail area is actually 2.5%, rather than 5%, as we have assumed. Applying Steps 1-4 for $p_0 = 0.95$ yields $a = 1.36$ and $b = 929.70 \times 10^3 h$. Table II gives the resultant BAZE test plan for the same set of values of γ and k used in Table I. Table III gives the percentage change in the BAZE plans of Table II relative to the "nominal" plans given in Table I. It is observed from Table III that the BAZE plans are somewhat sensitive to a 100% error (assumed 5% when, in fact, 2.5%) in the tail areas of the prior distribution. Now, if $a = 1.36$ and $b = 929.70 \times 10^3 h$, the "actual" posterior assurance of the plan $(nt)_0 =$ 1987 unit-hours is easily computed from (8) to be 75%, rather than 70% as required. Thus, we unknowingly would have more posterior assurance than required. Similarly, for any other BAZE test plan in Table I, we could compute the "true" posterior assurance relative to this "error" in fitting the prior distribution. Similarly, Tables IV and V consider the case where the actual tail areas of the gamma prior are 10%, instead of 5%, as assumed. From Table V, it is observed that the BAZE plans are fairly insensitive to this 100% error in the prior tail-area

TABLE II. Required Unit-Hours of Testing for Selected Valued of k and γ

$(a = 1.36,\ b = 929.70 \times 10^3 h)$

γ \ k	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.990	3466755.	2833963.	2381969.	2042971.	1779308.	1568377.	1395798.	1251982.	1130290.	1025984.	935586.
0.975	2618141.	2126785.	1775815.	1512558.	1307856.	1144070.	1010064.	898393.	803901.	722908.	652714.
0.950	1986835.	1600696.	1324832.	1118021.	957131.	828418.	723107.	635348.	561091.	497441.	442279.
0.900	1369130.	1085942.	883665.	731957.	613962.	519565.	442332.	377972.	323512.	276832.	236377.
0.850	1016911.	792426.	632080.	511819.	418284.	343456.	282232.	231213.	188043.	151040.	118971.
0.800	772714.	588929.	457653.	359196.	282619.	221357.	171234.	129465.	94121.	63826.	37572.
0.750	587545.	434621.	325390.	243466.	179747.	128773.	87066.	52311.	22902.	0.	0.
0.700	439706.	311422.	219790.	151066.	97615.	54853.	19867.	0.	0.	0.	0.
0.600	214683.	123903.	59059.	10427.	0.	0.	0.	0.	0.	0.	0.
0.500	49631.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE III. Percentage Change in the BAZE Plans of Table II

Relative to Table I

γ \ k	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	13.443%	12.468%	11.431%	10.326%	9.146%	7.883%	6.527%	5.068%	3.495%	1.792%	0.056%
.975	15.240	14.037	12.733	11.314	9.764	8.065	6.194	4.124	1.820	0.760	3.667
.950	17.068	15.595	13.958	12.128	10.069	7.735	5.066	1.986	1.609	5.860	10.964
.900	19.585	17.658	15.415	12.770	9.607	5.775	0.962	5.164	13.271	24.502	41.096
.850	21.602	19.217	16.290	12.614	7.858	1.465	7.587	21.392	45.026	94.782	267.49
.800	23.451	20.544	16.730	11.506	3.915	8.127	30.154	83.378	394.98		
.750	25.277	21.724	16.612	8.621	5.632	38.247	188.93				
.700	27.198	22.784	15.459	0.929	41.730	2660.6					
.600	32.155	24.254	8.377								
.500	46.718										

TABLE IV. Required Unit-Hours of Testing for Selected Values of k and γ

(a = 0.67, b = 330.68 x 10^3h)

γ \ k	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.990	2866755.	2233963.	1781969.	1442972.	1179308.	968378.	795798.	651983.	530291.	425985.	335536.
0.975	2018140.	1526785.	1175816.	912569.	707857.	544071.	410065.	298394.	203901.	122908.	52714.
0.950	1386834.	1000697.	724883.	518022.	357131.	228418.	123107.	35350.	0.	0.	0.
0.900	769131.	485943.	283665.	131957.	13962.	0.	0.	0.	0.	0.	0.
0.850	416911.	192426.	32080.	0.	0.	0.	0.	0.	0.	0.	0.
0.800	172714.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.750	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.700	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.600	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.500	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE V. Percentage Change in the BAZE Plans of Table IV Relative to Table I

γ \ k	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	28.424%	31.000%	33.741%	36.663%	39.783%	43.123%	46.707%	50.563%	54.723%	59.224%	64.111%
.975	34.664	38.288	42.218	46.493	51.161	56.280	61.917	68.156	75.098	82.869	91.628
.950	42.112	47.233	52.924	59.286	66.444	74.560	83.838	94.547			
.900	54.826	63.153	72.847	84.274	97.944						
.850	67.854	80.383	95.751								
.800	82.890										
.750											
.700											
.600											
.500											

probabilities. If $a = 0.67$ and $b = 330.68\times10^3$h, the actual posterior assurance of the plan $(nt)_0 = 1987$ unit-hours is 66%, as compared to the desired value of 70%. In this case, we have less posterior assurance than required.

Step 8: The unconditional probability of passing the test $(nt)_0 = 1987$ unit-hours is calculated from (10) as

$$P(\text{Passing the Test}) = [(0.60\times10^6)/(0.60\times10^6 + 1987)]$$
$$= 0.9967.$$

Table VI gives the unconditional probability of passing the corresponding test given in Table I. In practice, tables such as Table VI are useful to the producer in selecting the final test plan. From the results of Step 7 and this step, the final plan to be used is $(nt)_0 = 1987$ unit-hours of test.

Step 9: A single component may be tested for 1987 hours; five components for 397.4 hours; ten components for 198.7 hours; etc. If no failures occur, it may be claimed that a failure-rate of 2.0×10^{-6}f/h or less has been demonstrated with 70% assurance.

At this point the question can be raised; namely, how many unit-hours of testing are required when using an alternate classical (standard non-Bayesian) procedure? By judicious choice of producer and/or consumer risks, it is possible to "show" that classical test plans result in either larger or smaller total unit-hours of testing. Thus, a person advocating a purely classical approach could "show" that his procedure results in

TABLE VI

THE UNCONDITIONAL PROBABILITIES OF PASSING THE CORRESPONDING TESTS IN TABLE I

γ	k .5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	.130	.156	.182	.208	.235	.261	.287	.313	.339	.365	.391
.975	.163	.195	.228	.260	.293	.325	.358	.390	.423	.455	.488
.950	.200	.240	.280	.320	.361	.401	.441	.481	.521	.561	.601
.900	.261	.313	.365	.417	.469	.521	.573	.625	.677	.730	.782
.850	.316	.380	.443	.506	.569	.633	.696	.759	.822	.886	.949
.800	.373	.447	.522	.596	.671	.746	.820	.895	.969	1.000	1.000
.750	.433	.519	.606	.692	.779	.866	.952	1.000	1.000	1.000	1.000
.700	.498	.598	.698	.797	.897	.997	1.000	1.000	1.000	1.000	1.000
.600	.655	.786	.917	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.500	.866	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

less testing. On the other hand, a "Bayesian" can "show" the opposite to be the case. The correct answer lies in recognizing that the two procedures cannot be directly compared, due to the basic underlying philosophical difference regarding the interpretation of the failure-rate λ. In the classical approach, the failure-rate is a non-random (unknown) constant, while in the Bayesian approach it is an (unknown) value of a random variable. The proper procedure to be used should be based on this fact alone, and not on the basis of which procedure yields the smallest amount of testing. In some cases, the classical procedure will require less testing; in others, the BAZE procedure will require less. Since nuclear reactor components exhibit failure-rates which appear to be random, a Bayesian procedure, such as described here, should be used.

VI. PRIOR DISTRIBUTIONS AND ANALYSIS FOR SELECTED COMPONENTS OF NUCLEAR REACTOR SAFETY SYSTEMS

Table III 2-1 in Appendix III of the WASH 1400 Reactor Safety Study (1975) contains assessed estimates of the failure-rates of selected components of PWR and BWR safety systems. Upper and lower bounds, as well as the median, were computed from best available data for these failure-rates. The approach used there was also a Bayesian one and the random component failure-rates were assumed to follow a log-normal prior distribution. It is remarked here that the log-normal distribution is also a two-parameter positively skewed distribution similar to the gamma distribution used here. Although failure-rates were given on a per demand and per hour basis, depending upon the failure mode, only the time-dependent failure modes whose failure-rates are given on a per hour basis will be considered here. The upper and lower bounds were computed to be 5% bounds, respectively. That is, the probability that the given interval contains the failure-rate for any particular system is 90%. Thus $p_0 = 0.90$. These bounds are reproduced in Table VII for these same components and corresponding failure modes. If available, failure-rates corresponding to US nuclear operational experience are also given.

Gamma prior distributions have been fitted to these data by means of the procedure outlined in Section III. These prior distributions may be used in future test programs, such as those required in demonstrating the safety and reliability of planned LMFBR systems. The corresponding gamma parameters a and b, as well as the prior mean and variance, are given in Table VII. It is observed that there are only three different "shapes" of gamma priors present; namely, a = 0.50, 0.84, and 2.45. This is due to the fact that the ratio of UL to LL takes on only three different order of magnitude values; namely, 3, 2, or 1. The priors corresponding to a = 0.50 and 0.84 are quite diffuse; that is, the prior-variance is quite large for these priors.

Table VII also presents certain probabilities of interest associated with the corresponding fitted gamma prior distributions. The quantities P_1, P_2, P_3, P_4, and P_5 in Table VII represent the probabilities defined by

$P_1 \equiv$ Prob. ($\lambda \leq$ Prior Mean),

$P_2 \equiv$ Prob. ($\lambda \leq$ Median of a Log-Normal Prior),

$P_3 \equiv$ Prob. ($\lambda \leq$ US Nuclear Operating Experience Value),

$P_4 \equiv$ Prob. ($\lambda \leq$ Lower 5% Limit),

$P_5 \equiv$ Prob. ($\lambda \geq$ Upper 5% Limit).

These probabilities were calculated by means of the incomplete gamma function code, INCGAM, mentioned in Section III. Since the values of P_2 in Table VII are all less than 0.5, the gamma prior distributions tend to favor somewhat larger failure-rates than a log-normal distribution with the same 5% tail-area probabilities. The gamma approach is slightly conservative. Further, the values of P_4 and P_5 are included as a check on the accuracy of the procedure used to fit the gamma priors. The P_4 values are all 5%, to three decimal places, while the values of P_4 deviate somewhat from the desired value of 5% for a = 0.50 and 2.45. This departure is due to errors in reading and interpolating earlier, and slightly less accurate, versions of Figure A1 and Table A2, respectively.

VII. SUGGESTED BAZE TEST PLANS FOR SELECTED COMPONENTS OF NUCLEAR REACTOR SAFETY SYSTEMS

The purpose of this section is to propose several possible BAZE test plans for a few typical components selected from Table VII. It is emphasized that the plans given here are not

TABLE VII. Gamma Prior Distributions for Selected Components of Nuclear Power Reactor Safety Systems

Component	Failure Mode	Assessed Prior Failure Rates[1]: Lower 5% Limit(LL)	Median	Upper 5% Limit(UL)	US Nuclear Operating Experience[1]	a	b^2	Prior Mean[1]	Prior Variance[3]	P_1	P_2	P_3	P_4	P_5
Electric Clutch	Premature Open	$1x10^{-7}$	$1x10^{-6}$	$1x10^{-5}$	NA	0.84	$26.72x10^4$	$3.14x10^{-6}$	$1.18x10^{-11}$	0.64	0.31	NA	0.050	0.050
Instrumentation (Amplification, Annuciators, Transducers, Combination)	Failure to Operate	$1x10^{-7}$	$1x10^{-6}$	$1x10^{-5}$	$1x10^{-6}$	.84	$26.72x10^4$	$3.14x10^{-6}$	$1.18x10^{-11}$	.64	.31	.31	.050	.050
Mechanical Clutch	Failure to Open	$3x10^{-8}$	$3x10^{-7}$	$3x10^{-6}$	NA	.84	$89.08x10^4$	$9.43x10^{-7}$	$1.06x10^{-12}$	.64	.31	NA	.050	.050
Electric Motor	Failure to Run	$3x10^{-6}$	$1x10^{-5}$	$3x10^{-5}$	$1x10^{-6}$	2.45	$18.32x10^4$	$1.34x10^{-5}$	$7.30x10^{-11}$	.59	.42	.004	.050	.049
Electric Motor; Pumps	Failure to Run (Extreme Envir.)	$1x10^{-4}$	$1x10^{-3}$	$1x10^{-2}$	NA	.84	$26.72x10^1$	$3.15x10^{-3}$	$1.18x10^{-5}$	.64	.31	NA	.050	.050
Relays	Failure NO Contact to Close	$1x10^{-7}$	$3x10^{-7}$	$1x10^{-6}$	$1x10^{-6}$	2.45	$549.63x10^4$	$4.46x10^{-7}$	$8.11x10^{-14}$	.59	.36	.95	.050	.049
	Short Across NO/NC Contact	$1x10^{-9}$	$1x10^{-8}$	$1x10^{-7}$	$1x10^{-6}$	.84	$26.72x10^6$	$3.14x10^{-8}$	$1.18x10^{-15}$	.64	.31	1.00	.050	.050
	Open NC Contact	$3x10^{-8}$	$1x10^{-7}$	$3x10^{-7}$	$1x10^{-6}$	2.45	$18.32x10^6$	$1.34x10^{-7}$	$7.30x10^{-15}$	.59	.42	1.00	.050	.049

[1] (f/h) [2] (h) [3] (f^2/h^2)

TABLE VII. Gamma Prior Distributions for Selected Components of Nuclear Power Reactor Safety Systems (Continued)

Component	Failure Mode	Assessed Prior Failure Rates[1]: Lower 5% Limit(LL)	Median	Upper 5% Limit(UL)	US Nuclear Operating Experience[1]	a	b[2]	Prior Mean[1]	Prior Variance[3]	P_1	P_2	P_3	P_4	P_5
Switches;	Contacts Short	$1x10^{-9}$	$1x10^{-8}$	$1x10^{-7}$	$3x10^{-8}$	.84	$26.72x10^{6}$	$3.14x10^{-8}$	$1.18x10^{-15}$	0.64	0.31	0.63	0.050	0.050
Valves (MOV);	External Leak or Rupture													
Valves (AOV)	External Leak or Rupture													
Circuit Breakers;	Premature Transfer	$3x10^{-7}$	$1x10^{-6}$	$3x10^{-6}$	$1x10^{-6}$	2.45	$18.32x10^{5}$	$1.34x10^{-6}$	$7.30x10^{-13}$	.59	.42	.42	.050	.049
Transformers;	Open Circuit													
Transformers	Short													
Fuses	Premature Open	$3x10^{-7}$	$1x10^{-6}$	$3x10^{-6}$	NA	2.45	$18.32x10^{5}$	$1.34x10^{-6}$	$7.30x10^{-13}$	.59	.42	NA	.050	.049
Wires	Open	$1x10^{-6}$	$3x10^{-6}$	$1x10^{-5}$	$1x10^{-6}$	2.45	$54.96x10^{4}$	$4.46x10^{-6}$	$8.11x10^{-12}$	.59	.36	.05	.050	.049
	Short to GND	$3x10^{-8}$	$3x10^{-7}$	$3x10^{-6}$	$1x10^{-7}$	.84	$89.08x10^{4}$	$9.43x10^{-7}$	$1.06x10^{-12}$	.64	.31	.13	.050	.050
Wires; Valves (VACUUM); Valves (TEST)	Short to PWR Rupture Rupture	$1x10^{-9}$	$1x10^{-8}$	$1x10^{-7}$	NA	.84	$26.72x10^{6}$	$3.14x10^{-8}$	$1.18x10^{-15}$	.64	.31	NA	.050	.050

TABLE VII. Gamma Prior Distributions for Selected Components of Nuclear Power Reactor Safety Systems. (Continued)

Component	Failure Mode	Assessed Prior Failure Rates[1] Lower 5% Limit(LL)	Median	Upper 5% Limit(UL)	US Nuclear Operating Experience[1]	a	b^2	Prior Mean[1]	Prior Variance[3]	P_1	P_2	P_3	P_4	P_5
Solid State Devices	Fails to Function (Hi PWR Apps.)	$3x10^{-7}$	$3x10^{-6}$	$3x10^{-5}$	$1x10^{-6}$	.84	$89.08x10^{3}$	$9.43x10^{-6}$	$1.06x10^{-10}$	0.64	0 31	0.13	0.050	0.050
	Shorts (Hi PWR Apps.); Fails to Function (Low PWR Apps.)	$1x10^{-7}$	$1x10^{-6}$	$1x10^{-5}$	$1x10^{-7}$	.84	$26.72x10^{4}$	$3.14x10^{-6}$	$1.18x10^{-11}$	.64	.31	.05	.050	.050
	Shorts (Low PWR Apps.)	$1x10^{-8}$	$1x10^{-7}$	$1x10^{-6}$	NA	.84	$26.72x10^{5}$	$3.14x10^{-7}$	$1.18x10^{-13}$	.64	.31	NA	.050	.050
Pumps	Failure to Run (Normal Envir)	$3x10^{-6}$	$3x10^{-5}$	$3x10^{-4}$	$3x10^{-6}$	.84	$89.08x10^{2}$	$9.43x10^{-5}$	$1.06x10^{-8}$	.64	.31	.05	.050	.050
Valves (Check)	Reverse Leak	$1x10^{-7}$	$3x10^{-7}$	$1x10^{-6}$	NA	2.45	$549.63x10^{4}$	$4.46x10^{-7}$	$8.11x10^{-14}$	.59	.36	NA	.050	.049
	External Leaks-Ruptures	$1x10^{-9}$	$1x10^{-8}$	$1x10^{-7}$	$3x10^{-8}$	.84	$26.72x10^{6}$	$3.14x10^{-8}$	$1.18x10^{-15}$	.64	.31	.63	.050	.050

TABLE VII. Gamma Prior Distributions for Selected Components of Nuclear Power Reactor Safety Systems (Continued)

Components	Failure Mode	Assessed Prior Failure Rates[1] Lower 5% Limit (LL)	Median	Upper 5% Limit (UL)	US Nuclear Operating Experience[1]	a	b^2	Prior Mean[1]	Prior Variance[3]	P_1	P_2	P_3	P_4	P_5
Values (Relief)	Premature Open/Hr.	3×10^{-6}	1×10^{-5}	3×10^{-5}	1×10^{-5}	2.45	18.32×10^{4}	1.34×10^{-5}	7.30×10^{-11}	0.59	0.42	0.42	0.050	0.049
Pipes>3" (Hi Quality)	Rupture (Section)	3×10^{-12}	1×10^{-10}	3×10^{-9}	1×10^{-10}	.50	655.36×10^{6}	7.63×10^{-10}	1.16×10^{-18}	.68	.28	.28	.050	.047
Pipes<3"	Rupture	3×10^{-11}	1×10^{-9}	3×10^{-8}	1×10^{-9}	.50	655.36×10^{5}	7.63×10^{-9}	1.16×10^{-16}	.68	.28	.28	.050	.047
Gaskets	Leak	1×10^{-7}	3×10^{-6}	1×10^{-4}	1×10^{-6}	.50	19.66×10^{3}	2.54×10^{-9}	1.29×10^{-9}	.68	.27	.16	.050	.047
Flanges, Closures, Elbows	Leak/ Rupture	1×10^{-8}	3×10^{-7}	1×10^{-5}	NA	.50	19.66×10^{4}	2.54×10^{-6}	1.29×10^{-11}	.68	.27	NA	.050	.047
Welds	Leak	1×10^{-10}	3×10^{-9}	1×10^{-7}	NA	.50	19.66×10^{6}	2.54×10^{-8}	1.29×10^{-15}	.68	.27	NA	.050	.047
Diesel (Complete Plant)	Failure to Run	3×10^{-4}	3×10^{-3}	3×10^{-2}	1×10^{-3}	.84	89.08×10^{0}	9.43×10^{-3}	1.06×10^{-4}	.64	.31	.13	.050	.050
Diesel (Engine Only)	Failure to Run	3×10^{-5}	3×10^{-4}	3×10^{-3}	NA	.84	89.08×10^{1}	9.43×10^{-4}	1.06×10^{-6}	.64	.31	NA	.050	.050
Batteries, Power Supplies	NO/Output	1×10^{-6}	3×10^{-6}	1×10^{-5}	3×10^{-7}	2.45	54.96×10^{4}	4.46×10^{-6}	8.11×10^{-12}	.59	.36	.003	.050	.049
Instrumentation (Amplification, Annuciators, Transducers, Combination)	Shift Calibration	3×10^{-6}	3×10^{-5}	3×10^{-4}	1×10^{-4}	.84	89.08×10^{2}	9.43×10^{-5}	1.06×10^{-8}	.64	.31	.66	.050	.050

to be considered an exhaustive set or the sole source for the final selection. The final selection rests with the producer in conjunction with the consenting agreement of the consumer. It may be necessary or desirable to alter the prior distributions given in Table VII for numerous possible reasons. Such alterations will result in different test plans from those proposed here.

Before such BAZE test plans can be used in practice, several things must be considered. First, the life cycle of each component must be identified. This includes establishing the environments and operating conditions to which each component is exposed during the various phases of its existence. This also includes defining the duty cycle for the operational phases. Such practical details surrounding the implementation of these BAZE plans will not be addressed here. This would ordinarily be considered in the contractor's reliability program plan.

We shall restrict our consideration to three "typical" components given in Table VII; namely, electric clutchs ($a = 0.84$), electric motors ($a = 2.45$), and the category of flanges, closures, and elbows ($a = 0.50$). The specified failure-rate λ_0 will be taken to be the corresponding value of λ according to the US nuclear operating experience given in Table VII. If such values are not available, as indicated in Table VII, then λ_0 will be taken to be the assessed median prior failure-rate. Once a reliability, risk, or safety analysis program is under way, the value of λ_0 is a contract specification which may differ from the values considered here.

First, consider the electric clutch in Table VII. The BAZE procedure described in Section IV was applied to this component in conjunction with its fitted prior given in Table VII. Table VIII presents the resulting required unit-hours of testing for selected values of the discrimination ratio (k) and the posterior assurance (γ). For example, for k = 5 and γ = 0.90, it would require $(nt)_0 = [2.018547 - (26.72x10^4)(5)(1.0x10^{-6})]/[(5)(1.0x10^{-6})] =$ 136,510 unit-hours of electric clutch testing without a failure in order to demonstrate a failure-rate of $5.0x10^{-6}$f/h with 90% assurance. Table IX gives the unconditional probabilities of passing the corresponding tests given in Table VIII. The unconditional probability of passing the above test is 70.7%.

Similarly, Tables X and XI give the required BAZE unit-hours of testing and unconditional probabilities of passing the test for the case of an electric motor in a normal (nonextreme) environment. Likewise, Tables XII and XIII consider the category of flanges, closures, and elbows, with respect to leak or rupture failure modes.

Now let us consider the sensitivity of the BAZE procedure for each of these three component categories to errors in the tail-area probabilities. The procedure follows the example in Section V. Table XIV gives the fitted gamma prior distributions for these three components for actual prior probabilities of being

TABLE VIII. Required BAZE Unit-Hours of Testing for the Electric Clutch ($a = 0.84$, $b = 26.72 \times 10^4 h$)

	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
0.990	3.96090E 06	1.84685E 06	1.14217E 06	7.89826E 05	5.78421E 05	4.37484E 05	3.36815E 05	2.61313E 05	2.02589E 05	1.55610E 05	1.17173E 05
0.975	3.07549E 06	1.40414E 06	8.47029E 05	5.68472E 05	4.01337E 05	2.89914E 05	2.10327E 05	1.50636E 05	1.04210E 05	6.70686E 04	3.66806E 04
0.950	2.41041E 06	1.07161E 06	6.25337E 05	4.02203E 05	2.69322E 05	1.79069E 05	1.15316E 05	6.75014E 04	3.03123E 04	5.61142E 02	0.0
0.900	1.75135E 06	7.42073E 05	4.05649E 05	2.37437E 05	1.36510E 05	6.92245E 04	2.11639E 04	0.0	0.0	0.0	0.0
0.850	1.36985E 06	5.51323E 05	2.78482E 05	1.42062E 05	5.07095E 04	5.64114E 03	0.0	0.0	0.0	0.0	0.0
0.800	1.10169E 06	4.17247E 05	1.89098E 05	7.50237E 04	6.57902E 03	0.0	0.0	0.0	0.0	0.0	0.0
0.750	8.95584E 05	3.14192E 05	1.20395E 05	2.34962E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.700	7.28724E 05	2.30762E 05	5.47745E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600	4.69155E 05	1.00978E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	2.72162E 05	2.48113E 03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE IX. The Unconditional Probabilities of Passing the Corresponding Tests in Table VIII

	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
0.990	0.098	0.176	0.247	0.315	0.380	0.443	0.504	0.564	0.623	0.680	0.737
0.975	0.120	0.214	0.301	0.384	0.463	0.539	0.614	0.687	0.758	0.829	0.898
0.950	0.144	0.258	0.363	0.462	0.558	0.650	0.740	0.828	0.914	0.998	1.000
0.900	0.183	0.327	0.460	0.586	0.707	0.824	0.938	1.000	1.000	1.000	1.000
0.850	0.218	0.390	0.549	0.699	0.843	0.983	1.000	1.000	1.000	1.000	1.000
0.800	0.254	0.454	0.638	0.812	0.980	1.000	1.000	1.000	1.000	1.000	1.000
0.750	0.291	0.520	0.732	0.932	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.700	0.331	0.593	0.833	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.600	0.427	0.764	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.500	0.554	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE X. Required BAZE Unit-Hours of Testing for the Electric Motor (a = 2.45, b = 18.32x10^{4}h)

	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
0.990	[illegible].04490E 06	1.22617E 06	6.62421E 05	4.20815E 05	2.86589E 05	2.01173E 05	1.42039E 05	9.86736E 04	6.55120E 04	3.93319E 04	1.41383E 04
0.975	3.15949E 06	9.31029E 05	4.85337E 05	2.94327E 05	1.88210E 05	1.20681E 05	7.39296E 04	3.96457E 04	1.34286E 04	0.0	0.0
0.950	2.49441E 06	7.09337E 05	3.52322E 05	1.99316E 05	1.14312E 05	6.02192E 04	2.27701E 04	0.0	0.0	0.0	0.0
0.900	1.83535E 06	4.89649E 05	2.20509E 05	1.05164E 05	4.10830E 04	3.04309E 02	0.0	0.0	0.0	0.0	0.0
0.850	1.45385E 06	3.62482E 05	1.44209E 05	5.06638E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.800	1.18569E 06	2.73098E 05	9.05789E 04	1.23564E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.750	5.75584E 05	2.04395E 05	4.93569E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.700	8.12724E 05	1.48775E 05	1.59847E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600	5.53155E 05	4.22518E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	[illegible].56162E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XI. The Unconditional Probabilities of Passing the Corresponding Tests in Table X.

	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
0.990	0.000	0.007	0.024	0.054	0.100	0.163	0.245	0.348	0.473	0.621	0.794
0.975	0.001	0.012	0.042	0.096	0.177	0.289	0.436	0.619	0.841	1.000	1.000
0.950	0.001	0.021	0.072	0.165	0.305	0.498	0.750	1.000	1.000	1.000	1.000
0.900	0.003	0.041	0.144	0.329	0.609	0.996	1.000	1.000	1.000	1.000	1.000
0.850	0.005	0.069	0.241	0.550	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.800	0.007	0.107	0.374	0.852	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.750	0.011	0.159	0.557	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.700	0.016	0.233	0.815	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.600	0.033	0.488	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.500	0.071	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XII. Required BAZE Unit-Hours of Testing for Flanges, Closures, and Elbows ($a = 0.50$, $b = 19.66 \times 10^4 h$)

	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
0.990	1.08616E 07	2.56794E 06	1.38314E 06	9.09216E 05	6.54028E 05	4.94535E 05	3.85408E 05	3.06044E 05	2.45776E 05	1.98334E 05	1.6011[illegible] 05
0.975	8.17654E 06	1.89669E 06	9.94563E 05	6.40714E 05	4.47488E 05	3.26722E 05	2.44092E 05	1.83997E 05	1.38326E 05	1.02441E 05	7.3501[illegible] 04
0.950	6.20583E 06	1.40401E 06	7.18033E 05	4.43643E 05	2.95895E 05	2.03552E 05	1.40370E 05	9.44195E 04	5.94972E 04	3.20582E 04	9.93000E 03
0.900	4.31264E 06	9.30711E 05	4.47577E 05	2.54324E 05	1.50265E 05	8.52275E 04	4.07284E 04	8.36546E 03	0.0	0.0	0.0
0.850	3.25715E 06	5.66837E 05	2.96793E 05	1.48775E 05	6.90729E 04	1.92593E 04	0.0	0.0	0.0	0.0	0.0
0.800	2.54069E 06	4.87723E 05	1.94441E 05	7.71290E 04	1.39608E 04	0.0	0.0	0.0	0.0	0.0	0.0
0.750	2.00851E 06	3.54777E 05	1.18472E 05	2.39507E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.700	1.59372E 06	2.50981E 05	5.91605E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600	9.83943E 05	9.85359E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	5.61627E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XIII. The Unconditional Probabilities of Passing the Corresponding Tests in Table XII

	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
0.990	0.133	0.267	0.353	0.422	0.481	0.533	0.581	0.625	0.667	0.706	0.742
0.975	0.153	0.306	0.405	0.485	0.552	0.613	0.668	0.719	0.766	0.811	0.853
0.950	0.175	0.350	0.464	0.554	0.632	0.701	0.764	0.822	0.876	0.927	0.976
0.900	0.209	0.418	0.552	0.660	0.753	0.835	0.910	0.979	1.000	1.000	1.000
0.850	0.239	0.477	0.631	0.754	0.860	0.954	1.000	1.000	1.000	1.000	1.000
0.800	0.268	0.536	0.709	0.847	0.966	1.000	1.000	1.000	1.000	1.000	1.000
0.750	0.299	0.597	0.790	0.944	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.700	0.331	0.663	0.877	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.600	0.408	0.816	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.500	0.509	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

below and above LL and UL, respectively, the tail-area probabilities, of 2.5%, 5%, and 10%. Table XV gives the BAZE test plans for the electric clutch when the tail area probabilities are 2.5% instead of 5%. Table XVI gives the percentage change in the plans of Table XV relative to the "nominal" plans of Table VIII. Similarly, Tables XVII and XVIII correspond to Tables XV and XVI except that the tail-area probabilities are 10%. It is observed from Tables XVI and XVIII that the BAZE plans for the electric clutch are fairly sensitive to the tail-area probabilities in the gamma prior distribution. Similarly, Tables XIX-XXII and Tables XXIII-XXVI correspond to Tables XV-XVIII for the case of the electric motor and flanges, closures, and elbows, respectively. Again, the BAZE plans for these components are observed to be fairly sensitive to the tail-area probabilities.

Finally, similar BAZE test plans could be developed for the remaining components in Table VII. The BAZE procedure appears to be a practical means of conducting Bayesian reliability demonstration testing. The main advantage over other Bayesian procedures is its ease of application. A few simple tables and figures, which are included, and a pocket calculator are all that is required. The practical utility of the procedure appears to be evident.

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TABLE XIV. Gamma Prior Distributions for Selected Tail-Area Probabilities for the Electric Clutch, Electric Motor, and Flanges, Closures, and Elbows

Component	Tail-Area Probability (%)	P_0	a	b	Prior Mean	Prior Variance	P_1	P_2	P_3	P_4	P_5
Electric Clutch	2.5	0.95	1.13	4.10×10^5h	2.76×10^{-6}	6.72×10^{-12}	0.63	0.28	-	0.025	0.022
	5	.90	.84	26.72×10^4h	3.14×10^{-6}	1.18×10^{-11}	.64	.31	-	.050	.050
	10	.85	.60	18.87×10^4h	3.18×10^{-6}	1.69×10^{-10}	.67	.38	-	.103	.068
Electric Motor	2.5	.95	3.38	2.62×10^5h	1.29×10^{-5}	4.92×10^{-11}	.57	.40	0.001	.025	.024
	5	.90	2.45	18.32×10^4h	1.34×10^{-5}	7.30×10^{-11}	.59	.41	.004	.050	.049
	10	.85	1.59	1.11×10^5h	1.43×10^{-5}	1.29×10^{-10}	.61	.44	.020	.100	.095
Flanges, Closures, Elbows	2.5	.95	.67	34.93×10^4h	1.92×10^{-6}	5.49×10^{-12}	.66	.23	-	.025	.014
	5	.90	.50	19.66×10^4h	2.54×10^{-6}	1.29×10^{-11}	.68	.27	-	.050	.047
	10	.85	.35	100.34×10^3h	3.49×10^{-6}	3.48×10^{-11}	.71	.33	-	.100	.101

TABLE XV. Required BAZE Unit-Hours of Testing for the Electric Clutch ($a = 1.13$, $b = 4.10\times10^5$h)

γ \ k	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
0.990	4.48773E 06	2.03852E 06	1.22235E 06	8.14259E 05	5.69407E 05	4.06173E 05	2.89576E 05	2.02129E 05	1.34115E 05	7.97034E 04	3.51850E 04
0.975	3.54642E 06	1.56821E 06	9.08806E 05	5.79105E 05	3.81284E 05	2.49403E 05	1.55203E 05	8.45523E 04	2.96021E 04	0.0	0.0
0.950	2.83247E 06	1.21123E 06	6.73822E 05	4.00617E 05	2.38493E 05	1.30411E 05	5.32095E 04	0.0	0.0	0.0	0.0
0.900	2.11421E 06	8.52103E 05	4.31402E 05	2.21052E 05	9.48412E 04	1.07010E 04	0.0	0.0	0.0	0.0	0.0
0.850	1.69117E 06	6.40585E 05	2.90340E 05	1.15293E 05	1.52341E 04	0.0	0.0	0.0	0.0	0.0	0.0
0.800	1.38921E 06	4.89604E 05	1.89736E 05	3.98023E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.750	1.15363E 06	3.71016E 05	1.11211E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.750	9.60035E 05	2.75018E 05	4.66786E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600	6.51846E 05	1.20923E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	4.09573E 05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XVI. Percentage Change in the BAZE Plans of Table XV Relative to Table VIII

γ \ k	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
.990	3.605%	7.732%	12.503%	18.080%	24.688%	32.641%	42.397%	54.647%	70.487%	91.768%	
.975	4.643	10.170	16.859	25.120	35.581	49.256	67.894	94.798			
.950	5.924	13.326	22.836	35.504	53.220	79.746					
.900	8.154	19.243	35.203	60.142							
.850	10.424	25.901	51.278								
.800	12.962	34.224	75.516								
.750	15.945	45.450									
.700	19.596	61.882									
.600	30.438										
.500	52.469										

TABLE XVII. Required BAZE Unit-Hours of Testing for the Electric Clutch ($a = 0.60$, $b = 18.87 \times 10^4 h$)

γ \ k	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
0.990	4.70833E 06	2.25982E 06	1.44354E 06	1.03556E 06	7.99777E 05	6.27472E 05	5.10876E 05	4.23429E 05	3.55415E 05	3.01034E 05	2.56485E 05
0.975	3.76772E 06	1.78951E 06	1.13011E 06	8.00405E 05	6.52584E 05	4.70703E 05	3.76533E 05	3.05852E 05	2.50902E 05	2.06942E 05	1.70974E 05
0.950	3.35377E 06	1.43254E 06	8.92122E 05	6.21916E 05	4.59793E 05	3.51711E 05	2.74509E 05	2.16608E 05	1.71574E 05	1.35547E 05	1.06070E 05
0.900	2.33551E 06	1.07347E 06	6.52702E 05	4.42351E 05	3.16141E 05	2.32001E 05	1.71991E 05	1.26826E 05	9.17673E 04	6.37206E 04	4.57733E 04
0.850	1.91247E 06	8.61885E 05	5.11690E 05	3.36592E 05	2.31534E 05	1.61495E 05	1.11467E 05	7.39463E 04	4.47634E 04	2.14171E 04	2.31552E 03
0.800	1.61051E 06	7.12934E 05	4.11036E 05	2.61192E 05	1.71142E 05	1.11168E 05	6.83298E 04	3.62011E 04	1.12121E 04	0.0	0.0
0.750	1.37493E 06	5.93116E 05	3.32511E 05	2.02208E 05	1.24026E 05	7.19053E 04	3.46760E 04	6.75404E 03	0.0	0.0	0.0
0.700	1.18133E 06	4.96318E 05	2.67970E 05	1.53879E 05	8.53070E 04	3.96392E 04	7.01932E 03	0.0	0.0	0.0	0.0
0.600	8.73146E 05	3.42223E 05	1.65249E 05	7.67616E 04	2.36693E 04	0.0	0.0	0.0	0.0	0.0	0.0
0.500	6.30871E 05	2.21087E 05	8.44911E 04	1.61934E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XVIII. Percentage Change in the BAZE Plans of Table XVII Relative to Table VIII

γ \ k	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
.990	1.982%	4.250%	6.873%	9.938%	13.571%	17.944%	23.307%	30.041%	38.748%	50.447%	66.995%
.975	2.552	5.591	9.268	13.809	19.560	27.077	37.323	52.112	75.329	117.045	214.011
.950	3.257	7.325	12.553	19.518	29.256	43.838	68.074	116.293	258.907	13989.32	
.900	4.482	10.578	19.352	33.061	57.504	113.400	370.915				
.850	5.731	14.238	28.189	55.258	130.376	1391.562					
.800	7.125	18.814	41.513	104.634	1193.18						
.750	8.765	24.985	65.202	334.096							
.700	10.772	34.018	121.190								
.600	16.732	77.740									
.500	28.843	3163.88									

TABLE XIX. Required BAZE Unit-Hours of Testing for the Electric Motor ($a = 3.38$, $b = 2.62 \times 10^5 h$)

k	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
0.99[illegible]	4.635[illegible]3E [illegible]	1.37034E [illegible]	7.17407E [illegible]5	4.37576E 05	2.82115E 05	1.83185E 05	1.14695E 05	6.44690E 04	2.60609E 04	0.0	0.0
0.975	3.69442E 0[illegible]	1.[illegible]5681E 06	5.29294E [illegible]5	3.03203E 05	1.77602E 05	9.76744E 04	4.23399E 04	1.76125E 03	0.0	0.0	0.0
0.95[illegible]	2.98[illegible]47E [illegible]	8.18822E 05	3.86493E 05	2.01209E 05	9.32741E 04	3.27697E 04	0.0	0.0	0.0	0.0	0.0
0.9[illegible]	2.26221E [illegible]	5.794[illegible]2E [illegible]5	2.42641E [illegible]5	9.86008E 04	1.84674E 04	0.0	0.0	0.0	0.0	0.0	0.0
0.85[illegible]	1.83917E [illegible]	4.3839[illegible]E [illegible]5	1.58234E [illegible]	3.81672E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8[illegible]	[illegible].53721E 06	3.37736E 05	9.78419E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.75[illegible]	1.3[illegible]163E [illegible]	2.59211E [illegible]5	5.[illegible]7265E [illegible]4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7[illegible]	1.108[illegible]3E [illegible]	1.94678E [illegible]5	1.20071E [illegible]4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6[illegible]	7.99845E [illegible]5	[illegible].1[illegible]488E [illegible]4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5[illegible]	5.57573E [illegible]5	1.11912E [illegible]4	[illegible]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XX. Percentage Change in the BAZE Plans of Table XIX Relative to Table X

γ \ k	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
.990	1.958%	6.427%	11.896%	18.726%	27.496%	39.170%	55.478%	79.859%			
.975	2.494	8.464	16.236	26.773	41.868	65.297					
.950	3.159	11.109	22.366	39.536	68.934						
.900	4.293	16.093	35.735	74.931							
.850	5.420	21.739	54.643								
.800	6.646	28.854	86.996								
.750	8.044	38.553									
.700	9.696	52.966									
.600	14.246										
.500	22.125										

TABLE XXI. Required BAZE Unit-Hours of Testing for the Electric Motor ($a = 1.59$, $b = 1.11 \times 10^5 h$)

γ \ k	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
0.990	4.78603E 06	1.52134E 06	8.68407E 05	5.88576E 05	4.33115E 05	3.34185E 05	2.65695E 05	2.15469E 05	1.77061E 05	1.46739E 05	1.22192E 05
0.975	3.84542E 06	1.20781E 06	6.80284E 05	4.54203E 05	3.28602E 05	2.48674E 05	1.93340E 05	1.52761E 05	1.21731E 05	9.72326E 04	7.74009E 04
0.950	3.13147E 06	9.69822E 05	5.37493E 05	3.52209E 05	2.49274E 05	1.83770E 05	1.38420E 05	1.05164E 05	7.97333E 04	5.96561E 04	4.34032E 04
0.900	2.41321E 06	7.36402E 05	3.93841E 05	2.49601E 05	1.69467E 05	1.18473E 05	8.31597E 04	5.72804E 04	3.74827E 04	2.18530E 04	9.20028E 03
0.850	1.99017E 06	5.89390E 05	3.09234E 05	1.89167E 05	1.22463E 05	8.00154E 04	5.06285E 04	2.90780E 04	1.25983E 04	0.0	0.0
0.800	1.68821E 06	4.88736E 05	2.48842E 05	1.46030E 05	8.89121E 04	5.25645E 04	2.74007E 04	8.94725E 03	0.0	0.0	0.0
0.750	1.45263E 06	4.10211E 05	2.01726E 05	1.12376E 05	6.27369E 04	3.11484E 04	9.27940E 03	0.0	0.0	0.0	0.0
0.700	1.259[illegible]E 06	3.45678E 05	1.63007E 05	8.47193E 04	4.12261E 04	1.35487E 04	0.0	0.0	0.0	0.0	0.0
0.600	9.53446E 05	2.42949E 05	1.01369E 05	4.06923E 04	6.98293E 03	0.0	0.0	0.0	0.0	0.0	0.0
0.500	7.08574E 05	1.62191E 05	5.29147E 04	6.08192E 03	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XXII. Percentage Change in the BAZE Plans of Table XXI Relative to Table X

Y \ k	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0
.990	1.785%	5.888%	10.899%	17.157%	25.193%	35.8909%	50.831%	73.171%	110.209%	183.567%	398.053%
.975	2.285	7.755	14.876	24.530	38.361	59.821	97.661	182.114	537.658		
.950	2.894	10.179	20.493	36.224	63.160	119.895	317.082				
.900	3.934	14.745	32.742	68.655	175.742	23725.8					
.850	4.966	19.918	50.066	142.508							
.800	6.089	26.437	79.710	584.312							
.750	7.370	35.324	146.282								
.700	8.884	48.530	451.682								
.600	13.052	115.981									
.500	20.272										

TABLE XXII. Required BAZE Unit-Hours of Testing for the Flanges, Closures and Elbows ($a = 0.67$, $b = 34.93 \times 10^4 h$)

k	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
0.990	1.59741E 07	3.73156E 06	1.98252E 06	1.28304E 06	9.06350E 05	6.70915E 05	5.[illegible]9829E 05	3.92675E 05	3.[illegible]3638E 05	2.33680E 05	1.77263E 05
0.975	1.28388E 07	2.94772E 06	1.53471E 06	9.69506E 05	6.65166E 05	4.74954E 05	3.44808E 05	2.50157E 05	1.78222E 05	1.21702E 05	7.61211E 04
0.950	1.04589E 07	2.35276E 06	1.19473E 06	7.31522E 05	4.82101E 05	3.26213E 05	2.19554E 05	1.41983E 05	8.30287E 04	3.67077E 04	0.0
0.900	8.06471E 06	1.75420E 06	8.52702E 05	4.92102E 05	2.97932E 05	1.76576E 05	9.35431E 04	3.31554E 04	0.0	0.0	0.0
0.850	6.65459E 06	1.40167E 06	6.51257E 05	3.51090E 05	1.89461E 05	8.84436E 04	1.93263E 04	0.0	0.0	0.0	0.0
0.800	5.648[illegible]6E 06	1.150[illegible]4E 06	5.07466E 05	2.50436E 05	1.12035E 05	2.55350E 04	0.0	0.0	0.0	0.0	0.0
0.750	4.8628[illegible]E 06	9.53727E 05	3.95236E 05	1.71911E 05	5.16312E 04	0.0	0.0	0.0	0.0	0.0	0.0
0.700	4.21748E 06	7.92396E 05	3.03097E 05	1.07378E 05	1.99098E 03	0.0	0.0	0.0	0.0	0.0	0.0
0.600	3.19019E 06	5.35572E 05	1.56341E 05	4.64884E 03	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	2.38261E 06	3.33678E 05	4.09729E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XXIV. Percentage Change in the BAZE Plans of Table XXIII Relative to Table XII

γ \ k	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
.990	1.406%	5.946%	11.040%	16.795%	23.348%	30.877%	39.620%	49.895%	62.142%	76.991%	95.369%
.975	1.868	8.051	15.277	23.833	34.124	46.737	62.558	82.990			
.950	2.461	10.876	21.266	34.420	51.606	75.018					
.900	3.541	16.407	34.117	60.042	100.000	100.000					
.850	4.688	22.899	51.450	100.000							
.800	6.010	31.309	78.533								
.750	7.601	43.041	100.000								
.700	9.581	60.841									
.600	15.519	100.000									
.500	27.189										

TABLE XXV. Required BAZE Unit-Hours of Testing for the Flanges, Closures and Elbows ($a = 0.35$, $b = 100.34 \times 10^3 h$)

γ \ k	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
0.990	1.62231E 07	3.98052E 06	2.23158E 06	1.53200E 06	1.15531E 06	9.19875E 05	7.58788E 05	6.41635E 05	5.52598E 05	4.82640E 05	4.25223E 05
0.975	1.30877E 07	3.19668E 06	1.78367E 06	1.21847E 06	9.14126E 05	7.23913E 05	5.93768E 05	4.99117E 05	4.27182E 05	3.70662E 05	3.25081E 05
0.950	1.07379E 07	2.[illegible]172E 06	1.44359E 06	9.80481E 05	7.31061E 05	5.75173E 05	4.68513E 05	3.90943E 05	3.31989E 05	2.85668E 05	2.48312E 05
0.900	8.31367E 06	2.0[illegible]316E 06	1.10166E 06	7.41061E 05	5.46892E 05	4.25536E 05	3.42503E 05	2.82115E 05	2.36221E 05	2.00160E 05	1.71[illegible]E 05
0.850	6.90355E 06	1.65[illegible]64E 06	9.[illegible]7216E 05	6.00049E 05	4.38421E 05	3.37403E 05	2.68286E 05	2.18019E 05	1.79816E 05	1.49799E 05	1.25592E 05
0.800	5.897[illegible]2E 06	1.39[illegible]E 06	7.56426E 05	4.99396E 05	3.60995E 05	2.74495E 05	2.15310E 05	1.72267E 05	1.39554E 05	1.13851E 05	9.31232E 04
0.750	5.11176E 06	1.20264E 06	6.44246E 05	4.20870E 05	3.00591E 05	2.25416E 05	1.73981E 05	1.36574E 05	1.08144E 05	8.58066E 04	6.77924E 04
0.700	4.46644E 06	1.04136E 06	5.52057E 05	3.56338E 05	2.50951E 05	1.85084E 05	1.40017E 05	1.07241E 05	8.23313E 04	6.27593E 04	4.69755E 04
0.600	3.43914E 06	7.84532E 05	4.05301E 05	2.53609E 05	1.71928E 05	1.20878E 05	8.59488E 04	6.05458E 04	4.12395E 04	2.60702E 04	1.38370E 04
0.500	2.63157E 06	5.82638E 05	2.89933E 05	1.72851E 05	1.09807E 05	7.04044E 04	4.34448E 04	2.38378E 04	8.93642E 03	0.0	0.0

TABLE XXVI. Percentage Change in the BAZE Plans of Table XXV Relative to Table XII

γ \ k	1.0	4.0	7.0	10.0	13.0	16.0	19.0	22.0	25.0	28.0	31.0
.990	.886%	3.749%	6.960%	10.588%	14.718%	19.465%	24.976%	31.453%	39.174%	48.534%	60.119%
.975	1.177	5.075	9.630	15.024	21.511	29.462	39.436	52.316	69.589	93.966	130.963
.950	1.551	6.856	13.406	21.698	32.532	47.290	65.576	101.950	161.789	300.266	969.386
.900	2.232	10.343	21.507	37.849	64.060	112.945	236.345	1150.678			
.850	2.955	14.435	32.433	64.702	139.360	499.809					
.800	3.789	19.737	49.506	124.804	689.503						
.750	4.792	27.133	81.251	401.910							
.700	6.040	38.354	162.711								
.600	9.783	97.690									
.500	17.139										

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APPENDIX A. Some Useful Figures and Tables for the BAZE Procedure

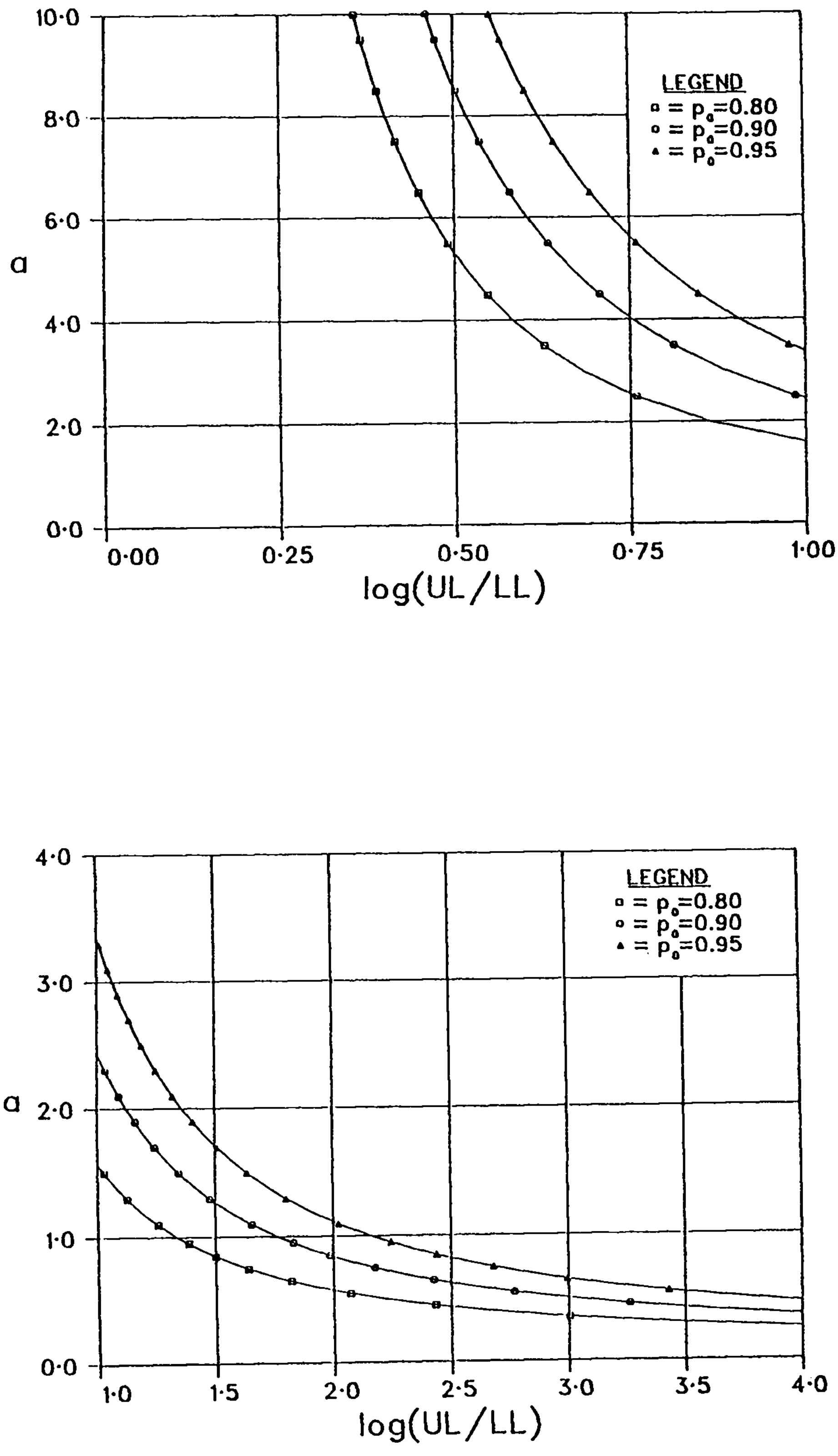

Fig. A1. Gamma Shape Parameter a as a Function of $\log_{10}$ (UL/LL).

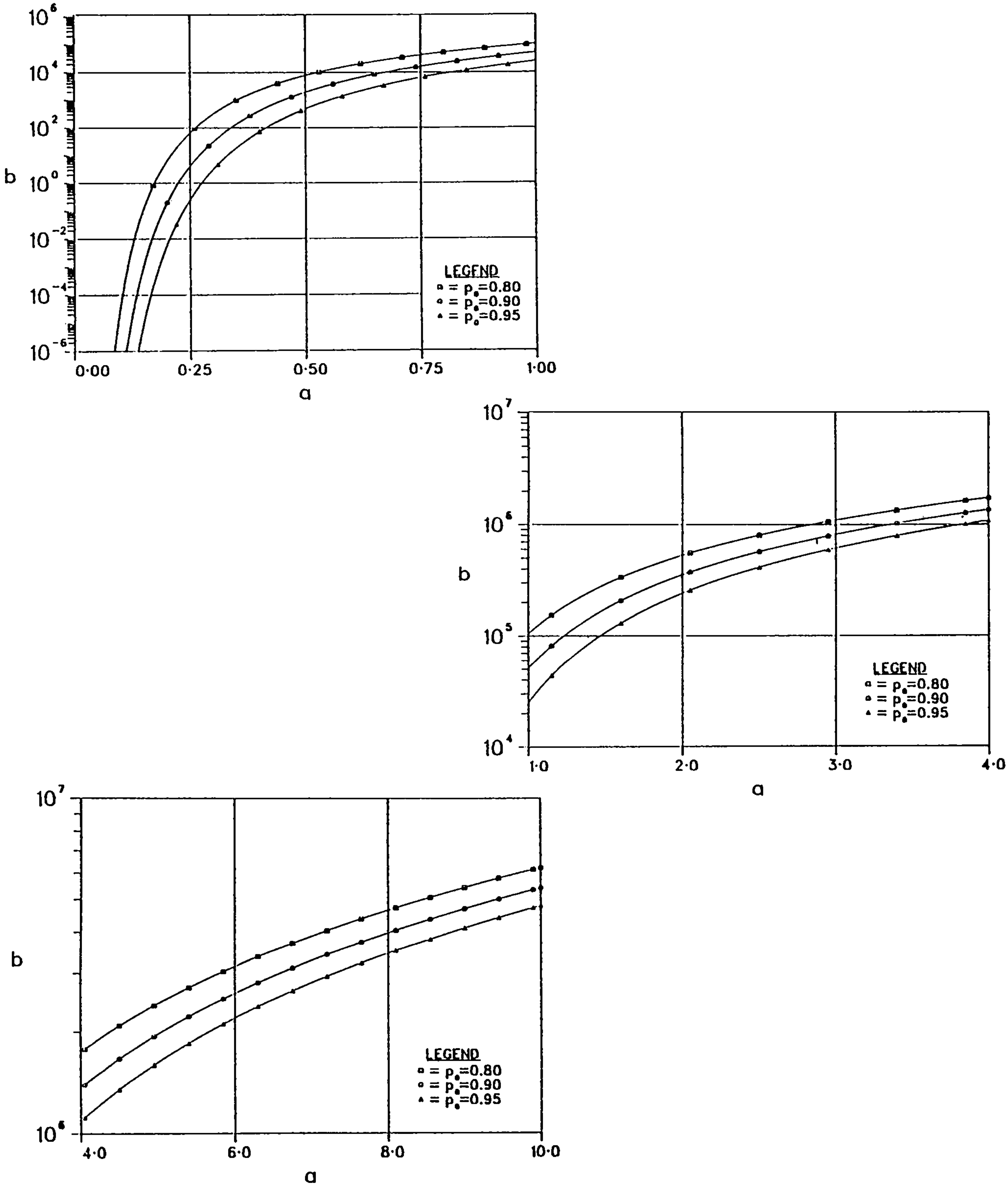

Fig. A2. Gamma Reference Scale Parameter b_0 as a Function of the Shape Parameter a.

Table A1. Values of θ_γ for Selected Values of Prior Shape Parameter (α) and Posterior Assurance (γ)

α	γ = 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.0001	1.2627E-44	6.2447-111	8.1261-149	7.6984-149	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0002	6.4210E-23	5.9220E-56	2.3303-112	7.6984-149	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0003	1.5952E-15	1.2530E-37	3.1245E-75	7.6984-149	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0004	6.8776E-12	1.8239E-28	1.1441E-56	2.2685-115	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0005	1.0469E-09	5.7376E-23	1.5729E-45	1.7160E-92	3.8668-142	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0006	2.9844E-08	2.6539E-19	4.1899E-38	3.0694E-77	1.3014-118	6.8431-149	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0007	3.2664E-07	1.1012E-16	8.4362E-33	2.4064E-66	8.3073-102	2.0262-139	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0008	1.9661E-06	1.0125E-14	8.9188E-29	3.5700E-58	3.3363E-89	4.0934-122	6.4154-149	5.9877-149	5.1323-149	4.2769-149
.0009	7.9407E-06	3.4084E-13	9.9561E-26	8.0907E-52	2.1195E-79	1.1799-108	8.4863-140	5.9877-149	5.1323-149	4.2769-149
.0010	2.4254E-05	5.6793E-12	2.9736E-23	9.8217E-47	1.4744E-71	6.9129E-98	6.4705-126	5.9877-149	5.1323-149	4.2769-149
.0011	6.0497E-05	5.6742E-11	3.1510E-21	1.4199E-42	3.8468E-65	4.4637E-89	1.4758-114	8.5064-142	5.1323-149	4.2769-149
.0012	1.2954E-04	3.8630E-10	1.5349E-19	4.1544E-39	8.5543E-60	9.8040E-82	4.3066-105	4.6215-130	5.1323-149	4.2769-149
.0013	2.4683E-04	1.9579E-09	4.1124E-18	3.5622E-36	2.8619E-55	1.5982E-75	4.3960E-97	3.9303-120	5.1323-149	4.2769-149
.0014	4.2883E-04	7.8700E-09	6.8882E-17	1.1638E-33	2.1615E-51	3.3758E-70	3.2201E-90	1.2751-111	5.1323-149	4.2769-149
.0015	6.9228E-04	2.6278E-08	7.9236E-16	1.7574E-31	4.9637E-48	1.3905E-65	2.8666E-84	3.0331-104	5.1323-149	4.2769-149
.0016	1.0528E-03	7.5470E-08	6.7165E-15	1.4173E-29	4.3323E-45	1.5175E-61	4.6045E-79	8.6331E-98	1.2429-139	4.2769-149
.0017	1.5262E-03	1.9145E-07	4.4276E-14	6.8195E-28	1.7047E-42	5.5467E-58	1.8053E-74	4.2769E-92	1.7810-131	4.2769-149
.0018	2.1183E-03	4.3794E-07	2.3669E-13	2.1337E-26	3.4534E-40	8.1483E-55	2.1855E-70	4.9353E-87	3.1671-124	4.2769-149
.0019	2.8642E-03	9.1623E-07	1.0607E-12	4.6463E-25	4.0000E-38	5.5555E-52	9.8348E-67	1.6697E-82	9.7162-118	4.2769-149
.0020	3.7884E-03	1.7679E-06	4.0910E-12	7.4391E-24	2.8807E-36	1.9725E-49	1.9084E-63	1.9909E-78	6.6936-112	4.2769-149
.0021	4.7166E-03	3.2672E-06	1.3876E-11	9.1366E-23	1.3805E-34	4.0035E-47	1.8007E-60	9.7107E-75	1.2818-106	2.5261-144
.0022	5.4700E-03	5.6520E-06	4.2118E-11	8.9381E-22	4.6537E-33	5.0130E-45	9.1153E-58	2.1884E-71	8.1230-102	8.2849-138
.0023	7.1697E-03	9.3227E-06	1.1608E-10	7.1710E-21	1.1554E-31	4.1243E-43	2.6850E-55	2.5203E-68	1.9686E-97	7.3711-132
.0024	8.6145E-03	1.4749E-05	2.9400E-10	4.8367E-20	2.1948E-30	2.3496E-41	4.9240E-53	1.6132E-65	2.0568E-93	2.0942-126
.0025	1.0202E-02	2.2494E-05	6.9128E-10	2.8004E-19	3.2944E-29	9.6874E-40	5.9527E-51	6.1578E-63	1.0250E-89	2.1788-121
.0026	1.1930E-02	3.3219E-05	1.5220E-09	1.4165E-18	4.0149E-28	3.0093E-38	4.9761E-49	1.4879E-60	2.6534E-86	9.3213-117
.0027	1.3792E-02	4.7636E-05	3.1606E-09	6.3542E-18	4.9658E-27	7.2058E-37	2.9969E-47	2.3943E-58	3.8378E-83	1.8101-112
.0028	1.5784E-02	6.6592E-05	6.2296E-09	2.5607E-17	3.4897E-26	1.3791E-35	1.3469E-45	2.6803E-56	3.3008E-80	1.7362-108
.0029	1.7905E-02	9.0966E-05	1.1718E-08	9.3730E-17	2.5826E-25	2.1533E-34	4.6562E-44	2.1671E-54	1.7815E-77	8.8492-105
.0030	2.0143E-02	1.2171E-04	2.1131E-08	3.1470E-16	1.6725E-24	2.7993E-33	1.2710E-42	1.3074E-52	6.3213E-75	2.5530-101
.0031	2.2494E-02	1.5981E-04	3.6685E-08	9.7711E-16	9.5014E-24	3.0841E-32	2.8029E-41	6.0541E-51	1.5357E-72	4.4053E-98
.0032	2.4957E-02	2.0630E-04	6.1530E-08	2.8265E-15	4.9417E-23	2.9247E-31	5.0945E-40	2.2060E-49	2.6468E-70	4.7707E-95
.0033	2.7521E-02	2.6223E-04	1.0002E-07	7.6664E-15	2.3030E-22	2.4201E-30	7.7673E-39	6.4441E-48	3.3389E-68	3.3829E-92
.0034	3.0181E-02	3.2865E-04	1.5800E-07	1.2699E-14	9.8037E-22	1.7684E-29	1.0089E-37	1.5529E-46	3.1688E-66	1.6304E-89
.0035	3.2032E-02	4.0663E-04	2.4315E-07	4.7533E-14	3.8419E-21	1.1534E-28	1.1318E-36	3.1107E-45	2.3185E-64	5.5207E-87
.0036	3.5768E-02	4.9721E-04	3.6535E-07	1.0970E-13	1.3956E-20	6.7788E-28	1.1102E-35	5.2757E-44	1.3364E-62	1.3526E-84
.0037	3.8685E-02	6.0141E-04	5.3702E-07	2.4196E-13	4.7279E-20	3.6293E-27	9.6252E-35	7.6779E-43	6.1875E-61	2.4614E-82
.0038	4.1676E-02	7.2022E-04	7.7351E-07	5.1195E-13	1.5021E-19	1.7703E-26	7.4483E-34	9.7051E-42	2.3411E-59	3.4061E-80
.0039	4.4737E-02	8.5457E-04	1.0935E-06	1.0424E-12	4.4978E-19	7.9797E-26	5.1896E-33	1.0771E-40	7.3521E-58	3.6604E-78
.0040	4.7963E-02	1.0054E-03	1.5193E-06	2.0482E-12	1.2749E-18	3.3361E-25	3.2814E-32	1.0599E-39	1.9434E-56	3.1135E-76
.0041	5.1050E-02	1.1735E-03	2.0774E-06	3.8942E-12	3.4347E-18	1.3007E-24	1.8963E-31	9.3286E-39	4.3785E-55	2.1322E-74
.0042	5.4293E-02	1.3597E-03	2.7984E-06	7.1808E-12	8.8267E-18	4.7534E-24	1.0081E-30	7.4028E-38	8.5052E-54	1.1940E-72
.0043	5.7589E-02	1.5648E-03	3.7179E-06	1.2870E-11	2.1709E-17	1.6355E-23	4.9583E-30	5.3351E-37	1.4392E-52	5.5447E-71
.0044	6.0931E-02	1.7894E-03	4.8761E-06	2.2402E-11	5.1255E-17	5.3197E-23	2.2684E-29	3.5148E-36	2.1415E-51	2.1626E-69
.0045	6.4319E-02	2.0341E-03	6.3185E-06	3.8240E-11	1.1648E-16	1.6420E-22	9.6997E-29	2.1294E-35	2.8262E-50	7.1675E-68
.0046	6.7747E-02	2.2996E-03	8.0959E-06	6.3632E-11	2.5541E-16	4.8257E-22	3.8937E-28	1.1029E-34	3.3340E-49	2.0401E-66
.0047	7.1213E-02	2.5862E-03	1.0265E-05	1.0360E-10	5.4168E-16	1.3547E-21	1.4733E-27	6.2105E-34	3.5410E-48	5.0357E-65
.0048	7.4713E-02	2.8945E-03	1.2886E-05	1.6520E-10	1.1134E-15	3.6429E-21	5.2738E-27	3.0185E-33	3.4083E-47	1.0875E-63
.0049	7.8245E-02	3.2248E-03	1.6029E-05	2.5870E-10	2.2221E-15	9.4083E-21	1.7921E-26	1.3754E-32	2.9910E-46	2.0719E-62
.0050	8.1806E-02	3.5774E-03	1.9762E-05	3.9774E-10	4.3140E-15	2.3394E-20	5.7990E-26	5.8980E-32	2.4063E-45	3.5083E-61

TABLE A1 (CONTINUED)

α	γ = 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.0051	.085393	.003953	2.4167E-05	6.0126E-10	8.1604E-15	5.6127E-20	1.7920E-25	2.3889E-31	1.7830E-44	5.3167E-60
.0052	.089003	.004351	2.9327E-05	8.9465E-10	1.5062E-14	1.3021E-19	5.3026E-25	9.1692E-31	1.2245E-43	7.2575E-59
.0053	.092635	.004772	3.5329E-05	1.3113E-09	2.7166E-14	2.9262E-19	1.5061E-24	3.3452E-30	7.8154E-43	8.9762E-58
.0054	.096286	.005216	4.2267E-05	1.8951E-09	4.7937E-14	6.3818E-19	4.1157E-24	1.1633E-29	4.6574E-42	1.0115E-56
.0055	.099954	.005683	5.0240E-05	2.7023E-09	8.2862E-14	1.3524E-18	1.0843E-23	3.8662E-29	2.6010E-41	1.0436E-55
.0056	.103638	.006173	5.9349E-05	3.8048E-09	1.4046E-13	2.7922E-18	2.7595E-23	1.2310E-28	1.3660E-40	9.9073E-55
.0057	.107335	.006687	6.9701E-05	5.2932E-09	2.3372E-13	5.6181E-18	6.7963E-23	3.7633E-28	6.7680E-40	8.6910E-54
.0058	.111043	.007223	8.1407E-05	7.2805E-09	3.8215E-13	1.1035E-17	1.6226E-22	1.1070E-27	3.1730E-39	7.0739E-53
.0059	.114761	.007783	9.4580E-05	9.9063E-09	6.1451E-13	2.1183E-17	3.7615E-22	3.1394E-27	1.4123E-38	5.3627E-52
.0060	.118489	.008366	1.0934E-04	1.3342E-08	9.7262E-13	3.9791E-17	8.4788E-22	8.5093E-27	5.9795E-38	3.8000E-51
.0061	.122223	.008971	1.2580E-04	1.7794E-08	1.5164E-12	7.3216E-17	1.8609E-21	2.2789E-26	2.4146E-37	2.5253E-50
.0062	.125963	.009599	1.4409E-04	2.3512E-08	2.3307E-12	1.3209E-16	3.9821E-21	5.8525E-26	9.3212E-37	1.5787E-49
.0063	.129708	.010249	1.6432E-04	3.0794E-08	3.5336E-12	2.3389E-16	8.3180E-21	1.4586E-25	3.4473E-36	9.3116E-49
.0064	.133456	.010921	1.8663E-04	3.9994E-08	5.2882E-12	4.0683E-16	1.6979E-20	3.5332E-25	1.2230E-35	5.1959E-48
.0065	.137207	.011615	2.1115E-04	5.1525E-08	7.8164E-12	6.9566E-16	3.3907E-20	8.3284E-25	4.1788E-35	2.7500E-47
.0066	.140959	.012331	2.3799E-04	6.5874E-08	1.1417E-11	1.1704E-15	6.6307E-20	1.9128E-24	1.3747E-34	1.3938E-46
.0067	.144711	.013068	2.6729E-04	8.3604E-08	1.6490E-11	1.9386E-15	1.2710E-19	4.2856E-24	4.3647E-34	6.6352E-46
.0068	.148463	.013827	2.9918E-04	1.0537E-07	2.3560E-11	3.1642E-15	2.3900E-19	9.3765E-24	1.3394E-33	3.0392E-45
.0069	.152214	.014606	3.3378E-04	1.3190E-07	3.3315E-11	5.0015E-15	4.4128E-19	2.0055E-23	3.9791E-33	1.3312E-44
.0070	.155963	.015405	3.7122E-04	1.6407E-07	4.6644E-11	8.0821E-15	8.0061E-19	4.1072E-23	1.1459E-32	5.5915E-44
.0071	.159708	.016224	4.1163E-04	2.0283E-07	6.4692E-11	1.2663E-14	1.4284E-18	8.6034E-23	3.2020E-32	2.2556E-43
.0072	.163450	.017064	4.5513E-04	2.4927E-07	8.8911E-11	1.9590E-14	2.5077E-18	1.7287E-22	8.7007E-32	8.7531E-43
.0073	.167189	.017922	5.0185E-04	3.0463E-07	1.2114E-10	2.9962E-14	4.3353E-18	3.4078E-22	2.2997E-31	3.2729E-42
.0074	.170922	.018800	5.5191E-04	3.7026E-07	1.6367E-10	4.5290E-14	7.3848E-18	6.5056E-22	5.9210E-31	1.1809E-41
.0075	.174650	.019696	6.0543E-04	4.4770E-07	2.1937E-10	6.7710E-14	1.2402E-17	1.2543E-21	1.4865E-30	4.1177E-41
.0076	.178372	.020611	6.6253E-04	5.3864E-07	2.9177E-10	1.0016E-13	2.0545E-17	2.3452E-21	3.6425E-30	1.3894E-40
.0077	.182088	.021544	7.2332E-04	6.4495E-07	3.8520E-10	1.4667E-13	3.3593E-17	4.3144E-21	8.7203E-30	4.5421E-40
.0078	.185797	.022494	7.8793E-04	7.6869E-07	5.0494E-10	2.1268E-13	5.4238E-17	7.8139E-21	2.0415E-29	1.4405E-39
.0079	.189498	.023462	8.5645E-04	9.1210E-07	6.5738E-10	3.0552E-13	8.6517E-17	1.3041E-20	4.6775E-29	4.4368E-39
.0080	.193192	.024446	9.2900E-04	1.0777E-06	8.5022E-10	4.3492E-13	1.3640E-16	2.4514E-20	1.0497E-28	1.3287E-38
.0081	.196878	.025447	1.0057E-03	1.2630E-06	1.0927E-09	6.1376E-13	2.1265E-16	4.2511E-20	2.3092E-28	3.8726E-38
.0082	.200556	.026464	1.0866E-03	1.4801E-06	1.3957E-09	8.5890E-13	3.2795E-16	7.2737E-20	4.9832E-28	1.0997E-37
.0083	.204225	.027497	1.1719E-03	1.7351E-06	1.7723E-09	1.1922E-12	5.0051E-16	1.2285E-19	1.0556E-27	3.0451E-37
.0084	.207885	.028546	1.2615E-03	2.0183E-06	2.2377E-09	1.6421E-12	7.5672E-16	2.0493E-19	2.1966E-27	8.2300E-37
.0085	.211536	.029609	1.3557E-03	2.3374E-06	2.8099E-09	2.2447E-12	1.1315E-15	3.3774E-19	4.4025E-27	2.1729E-36
.0086	.215178	.030687	1.4545E-03	2.7023E-06	3.5097E-09	3.0463E-12	1.6773E-15	5.5020E-19	9.0367E-27	5.6091E-36
.0087	.218809	.031780	1.5580E-03	3.1112E-06	4.3616E-09	4.1052E-12	2.4640E-15	8.8432E-19	1.7888E-26	1.4167E-35
.0088	.222431	.032886	1.6663E-03	3.5705E-06	5.3935E-09	5.4947E-12	3.5881E-15	1.4124E-18	3.4863E-26	3.5034E-35
.0089	.226042	.034007	1.7794E-03	4.0850E-06	6.6378E-09	7.3066E-12	5.1811E-15	2.2273E-18	6.6936E-26	8.4896E-35
.0090	.229643	.035140	1.8975E-03	4.6597E-06	8.1316E-09	9.6547E-12	7.4206E-15	3.4769E-18	1.2667E-25	2.0172E-34
.0091	.233234	.036287	2.0205E-03	5.2998E-06	9.9173E-09	1.2680E-11	1.0544E-14	5.3748E-18	2.3636E-25	4.7026E-34
.0092	.236814	.037446	2.1487E-03	6.0111E-06	1.2043E-08	1.6554E-11	1.4869E-14	8.2304E-18	4.3511E-25	1.0763E-33
.0093	.240383	.038618	2.2819E-03	6.7994E-06	1.4563E-08	2.1488E-11	2.0814E-14	1.2486E-17	7.9054E-25	2.4200E-33
.0094	.243941	.039801	2.4203E-03	7.6799E-06	1.7540E-08	2.7739E-11	2.8927E-14	1.8781E-17	1.4182E-24	5.3490E-33
.0095	.247488	.040997	2.5640E-03	8.6322E-06	2.1043E-08	3.5616E-11	3.9925E-14	2.8004E-17	2.5130E-24	1.1623E-32
.0096	.251023	.042203	2.7129E-03	9.6901E-06	2.5150E-08	4.5492E-11	5.4737E-14	4.1411E-17	4.4003E-24	2.4857E-32
.0097	.254548	.043421	2.8672E-03	1.0852E-05	2.9948E-08	5.7815E-11	7.4556E-14	6.0742E-17	7.6166E-24	5.2330E-32
.0098	.258061	.044650	3.0269E-03	1.2125E-05	3.5535E-08	7.3118E-11	1.0091E-13	8.8406E-17	1.3037E-23	1.0851E-31
.0099	.261563	.045889	3.1920E-03	1.3517E-05	4.2018E-08	9.2034E-11	1.3576E-13	1.2770E-16	2.2074E-23	2.2170E-31
.0100	.265053	.047138	3.3626E-03	1.5036E-05	4.9518E-08	1.1531E-10	1.8155E-13	1.8310E-16	3.6983E-23	4.4655E-31

Table A1 (continued)

a	γ = 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.0110	.299307	.060131	5.3753E-03	3.9217E-05	2.1715E-07	8.7746E-10	2.4840E-12	4.6906E-15	3.8473E-21	2.4370E-28
.0120	.332376	.073864	7.9546E-03	8.7195E-05	7.4443E-07	4.7615E-09	2.1980E-11	6.9998E-14	1.8458E-19	4.6531E-26
.0130	.364275	.088128	1.1094E-02	1.7147E-04	2.1116E-06	1.9921E-08	1.3908E-10	6.8928E-13	4.8825E-18	3.9608E-24
.0140	.395042	.102761	1.4771E-02	3.0621E-04	5.1616E-06	6.7942E-08	6.7622E-10	4.8962E-12	8.0900E-17	1.7871E-22
.0150	.424727	.117634	1.8950E-02	5.0624E-04	1.1200E-05	1.9678E-07	2.6631E-09	2.6782E-11	9.2198E-16	4.8526E-21
.0160	.453386	.132646	2.3591E-02	7.8611E-04	2.2064E-05	4.9992E-07	8.8373E-09	1.1847E-10	7.7525E-15	8.7220E-20
.0170	.481076	.147718	2.8649E-02	1.1594E-03	4.0136E-05	1.1344E-06	2.5469E-08	4.4001E-10	5.0747E-14	1.1161E-18
.0180	.507854	.162790	3.4083E-02	1.6389E-03	6.8320E-05	2.3540E-06	6.5261E-08	1.4126E-09	2.6963E-13	1.0760E-17
.0190	.533774	.177815	3.9849E-02	2.2341E-03	1.0998E-04	4.5240E-06	1.5147E-07	4.0115E-09	1.2017E-12	8.1726E-17
.0200	.558886	.192756	4.5910E-02	2.9497E-03	1.6882E-04	8.1451E-06	3.2319E-07	1.0264E-08	4.6127E-12	5.0687E-16
.0210	.583237	.207587	5.2228E-02	3.7959E-03	2.4880E-04	1.3867E-05	6.4162E-07	2.4014E-08	1.5578E-11	2.6423E-15
.0220	.606873	.222286	5.8770E-02	4.7779E-03	3.5401E-04	2.2496E-05	1.1969E-06	5.2011E-08	4.7104E-11	1.1855E-14
.0230	.629835	.236837	6.5506E-02	5.8950E-03	4.8855E-04	3.4993E-05	2.1151E-06	1.0534E-07	1.2937E-10	4.6686E-14
.0240	.652160	.251230	7.2409E-02	7.1472E-03	6.5644E-04	5.2468E-05	3.5646E-06	2.0116E-07	3.2666E-10	1.6401E-13
.0250	.673884	.265454	7.9454E-02	8.5399E-03	8.6152E-04	7.6168E-05	5.7623E-06	3.6481E-07	7.6597E-10	5.2112E-13
.0260	.695041	.279506	8.6620E-02	1.0066E-02	1.1074E-03	1.0746E-04	8.9775E-06	6.3201E-07	1.6821E-09	1.5150E-12
.0270	.715660	.293382	9.3888E-02	1.1724E-02	1.3974E-03	1.4779E-04	1.3536E-05	1.0513E-06	3.4851E-09	4.0699E-12
.0280	.735770	.307079	.101240	1.3511E-02	1.7345E-03	1.9870E-04	1.9820E-05	1.6865E-06	6.8551E-09	1.0189E-11
.0290	.755396	.320597	.108662	1.5423E-02	2.1213E-03	2.6177E-04	2.8269E-05	2.6187E-06	1.2870E-08	2.3944E-11
.0300	.774564	.333936	.116138	1.7457E-02	2.5601E-03	3.3860E-04	3.9380E-05	3.9490E-06	2.3170E-08	5.3155E-11
.0310	.793296	.347097	.123659	1.9606E-02	3.0528E-03	4.3080E-04	5.3699E-05	5.7995E-06	4.0162E-08	1.1209E-10
.0320	.811612	.360083	.131212	2.1869E-02	3.6011E-03	5.3996E-04	7.1824E-05	8.3156E-06	6.7267E-08	2.2562E-10
.0330	.829533	.372895	.138788	2.4236E-02	4.2060E-03	6.6793E-04	9.4393E-05	1.1666E-05	1.0920E-07	4.3531E-10
.0340	.847076	.385536	.146381	2.6706E-02	4.8685E-03	8.1529E-04	1.2208E-04	1.6045E-05	1.7231E-07	8.0806E-10
.0350	.864259	.398009	.153981	2.9272E-02	5.5892E-03	9.8439E-04	1.5560E-04	2.1670E-05	2.6490E-07	1.4490E-09
.0360	.881098	.410317	.161582	3.1930E-02	6.3685E-03	1.1763E-03	1.9567E-04	2.8783E-05	3.9764E-07	2.5120E-09
.0370	.897607	.422463	.169179	3.4675E-02	7.2064E-03	1.3922E-03	2.4304E-04	3.7651E-05	5.8397E-07	4.2302E-09
.0380	.913801	.434451	.176768	3.7502E-02	8.1028E-03	1.6333E-03	2.9848E-04	4.8562E-05	8.4047E-07	6.9311E-09
.0390	.929693	.446284	.184342	4.0407E-02	9.0573E-03	1.9007E-03	3.6273E-04	6.1826E-05	1.1873E-06	1.1073E-08
.0400	.945295	.457965	.191899	4.3384E-02	1.0069E-02	2.1953E-03	4.3656E-04	7.7771E-05	1.6486E-06	1.7282E-08
.0410	.960618	.469498	.199435	4.6430E-02	1.1138E-02	2.5181E-03	5.2073E-04	9.6744E-05	2.2529E-06	2.6393E-08
.0420	.975675	.480886	.206947	4.9541E-02	1.2264E-02	2.8697E-03	6.1595E-04	1.1910E-04	3.0333E-06	3.9504E-08
.0430	.990474	.492133	.214433	5.2713E-02	1.3444E-02	3.2508E-03	7.2296E-04	1.4523E-04	4.0280E-06	5.8032E-08
.0440	1.005026	.503242	.221890	5.5942E-02	1.4678E-02	3.6621E-03	8.4244E-04	1.7550E-04	5.2807E-06	8.3775E-08
.0450	1.019340	.514216	.229316	5.9223E-02	1.5966E-02	4.1039E-03	9.7508E-04	2.1032E-04	6.8404E-06	1.1899E-07
.0460	1.033425	.525058	.236710	6.2555E-02	1.7305E-02	4.5767E-03	1.1215E-03	2.5007E-04	8.7619E-06	1.6644E-07
.0470	1.047288	.535772	.244069	6.5933E-02	1.8695E-02	5.0807E-03	1.2823E-03	2.9517E-04	1.1106E-05	2.2953E-07
.0480	1.060938	.546361	.251394	6.9355E-02	2.0135E-02	5.6162E-03	1.4581E-03	3.4601E-04	1.3930E-05	3.1233E-07
.0490	1.074382	.556827	.258682	7.2817E-02	2.1622E-02	6.1833E-03	1.6494E-03	4.0301E-04	1.7334E-05	4.1971E-07
.0500	1.087627	.567174	.265932	7.6317E-02	2.3156E-02	6.7820E-03	1.8567E-03	4.6656E-04	2.1369E-05	5.5739E-07
.0510	1.100680	.577404	.273145	7.9852E-02	2.4735E-02	7.4124E-03	2.0805E-03	5.3706E-04	2.6130E-05	7.3206E-07
.0520	1.113547	.587520	.280319	8.3420E-02	2.6359E-02	8.0744E-03	2.3213E-03	6.1490E-04	3.1705E-05	9.5148E-07
.0530	1.126235	.597525	.287454	8.7018E-02	2.8025E-02	8.7679E-03	2.5793E-03	7.0047E-04	3.8192E-05	1.2245E-06
.0540	1.138747	.607422	.294549	9.0644E-02	2.9732E-02	9.4957E-03	2.8550E-03	7.9413E-04	4.5690E-05	1.5613E-06
.0550	1.151092	.617213	.301605	9.4295E-02	3.1480E-02	1.0249E-02	3.1488E-03	8.9624E-04	5.4308E-05	1.9733E-06
.0560	1.163273	.626900	.308620	9.7971E-02	3.3266E-02	1.1035E-02	3.4607E-03	1.0072E-03	6.4156E-05	2.4732E-06
.0570	1.175294	.636486	.315595	.101668	3.5089E-02	1.1853E-02	3.7913E-03	1.1272E-03	7.5350E-05	3.0754E-06
.0580	1.187163	.645973	.322530	.105386	3.6948E-02	1.2700E-02	4.1405E-03	1.2568E-03	8.8010E-05	3.7958E-06
.0590	1.198882	.655364	.329425	.109122	3.8843E-02	1.3577E-02	4.5087E-03	1.3961E-03	1.0226E-04	4.6517E-06
.0600	1.210456	.664660	.336278	.112876	4.0770E-02	1.4484E-02	4.8960E-03	1.5455E-03	1.1823E-04	5.6622E-06

TABLE A1 (CONTINUED)

a	γ = 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.0610	1.221890	.673865	.343092	.116644	4.2731E-02	1.5420E-02	5.3025E-03	1.7053E-03	1.3604E-04	6.8481E-06
.0620	1.233187	.682979	.349866	.120427	4.4722E-02	1.6384E-02	5.7282E-03	1.8757E-03	1.5584E-04	8.2320E-06
.0630	1.244351	.692005	.356599	.124222	4.6743E-02	1.7377E-02	6.1734E-03	2.0570E-03	1.7776E-04	9.8381E-06
.0640	1.255386	.700944	.363293	.128028	4.8794E-02	1.8397E-02	6.6379E-03	2.2494E-03	2.0193E-04	1.1692E-05
.0650	1.266295	.709800	.369947	.131845	5.0872E-02	1.9445E-02	7.1219E-03	2.4532E-03	2.2849E-04	1.3823E-05
.0660	1.277081	.718573	.376562	.135671	5.2976E-02	2.0519E-02	7.6253E-03	2.6684E-03	2.5759E-04	1.6260E-05
.0670	1.287749	.727265	.383137	.139504	5.5107E-02	2.1619E-02	8.1481E-03	2.8954E-03	2.8936E-04	1.9034E-05
.0680	1.298300	.735878	.389674	.143345	5.7262E-02	2.2745E-02	8.6902E-03	3.1343E-03	3.2396E-04	2.2179E-05
.0690	1.308739	.744414	.396172	.147192	5.9441E-02	2.3897E-02	9.2516E-03	3.3853E-03	3.6151E-04	2.5729E-05
.0700	1.319067	.752874	.402632	.151044	6.1643E-02	2.5073E-02	9.8322E-03	3.6484E-03	4.0216E-04	2.9723E-05
.0710	1.329288	.761260	.409054	.154900	6.3866E-02	2.6273E-02	1.0432E-02	3.9239E-03	4.4606E-04	3.4198E-05
.0720	1.339404	.769574	.415438	.158760	6.6111E-02	2.7496E-02	1.1050E-02	4.2118E-03	4.9334E-04	3.9194E-05
.0730	1.349417	.777816	.421785	.162623	6.8375E-02	2.8743E-02	1.1688E-02	4.5122E-03	5.4413E-04	4.4754E-05
.0740	1.359332	.785988	.428095	.166488	7.0659E-02	3.0013E-02	1.2344E-02	4.8253E-03	5.9850E-04	5.0920E-05
.0750	1.369148	.794093	.434369	.170354	7.2962E-02	3.1304E-02	1.3019E-02	5.1510E-03	6.5684E-04	5.7739E-05
.0760	1.378870	.802130	.440607	.174221	7.5282E-02	3.2617E-02	1.3712E-02	5.4896E-03	7.1901E-04	6.5255E-05
.0770	1.388499	.810102	.446809	.178088	7.7619E-02	3.3951E-02	1.4423E-02	5.8409E-03	7.8525E-04	7.3517E-05
.0780	1.398037	.818009	.452975	.181955	7.9972E-02	3.5306E-02	1.5152E-02	6.2051E-03	8.5566E-04	8.2575E-05
.0790	1.407486	.825854	.459106	.185821	8.2341E-02	3.6681E-02	1.5899E-02	6.5821E-03	9.3040E-04	9.2477E-05
.0800	1.416849	.833636	.465203	.189686	8.4725E-02	3.8075E-02	1.6664E-02	6.9721E-03	1.0096E-03	1.0328E-04
.0810	1.426127	.841358	.471266	.193549	8.7123E-02	3.9489E-02	1.7446E-02	7.3749E-03	1.0933E-03	1.1503E-04
.0820	1.435323	.849020	.477294	.197410	8.9534E-02	4.0921E-02	1.8245E-02	7.7906E-03	1.1817E-03	1.2778E-04
.0830	1.444437	.856624	.483289	.201268	9.1959E-02	4.2371E-02	1.9061E-02	8.2192E-03	1.2749E-03	1.4159E-04
.0840	1.453472	.864171	.489251	.205123	9.4396E-02	4.3839E-02	1.9894E-02	8.6607E-03	1.3729E-03	1.5651E-04
.0850	1.462429	.871661	.495181	.208974	9.6844E-02	4.5325E-02	2.0744E-02	9.1150E-03	1.4760E-03	1.7260E-04
.0860	1.471311	.879096	.501077	.212822	9.9304E-02	4.6827E-02	2.1610E-02	9.5820E-03	1.5842E-03	1.8991E-04
.0870	1.480119	.886477	.506942	.216666	.101775	4.8345E-02	2.2491E-02	1.0062E-02	1.6976E-03	2.0851E-04
.0880	1.488852	.893805	.512775	.220506	.104256	4.9880E-02	2.3389E-02	1.0554E-02	1.8163E-03	2.2844E-04
.0890	1.497515	.901080	.518577	.224341	.106746	5.1429E-02	2.4302E-02	1.1060E-02	1.9404E-03	2.4977E-04
.0900	1.506107	.908304	.524348	.228171	.109246	5.2994E-02	2.5230E-02	1.1577E-02	2.0700E-03	2.7256E-04
.0910	1.514632	.915477	.530088	.231996	.111755	5.4574E-02	2.6174E-02	1.2107E-02	2.2052E-03	2.9687E-04
.0920	1.523089	.922601	.535799	.235815	.114272	5.6168E-02	2.7133E-02	1.2650E-02	2.3460E-03	3.2274E-04
.0930	1.531480	.929676	.541479	.239629	.116797	5.7776E-02	2.8106E-02	1.3205E-02	2.4925E-03	3.5025E-04
.0940	1.539807	.936703	.547130	.243437	.119330	5.9397E-02	2.9093E-02	1.3772E-02	2.6449E-03	3.7945E-04
.0950	1.548070	.943684	.552752	.247239	.121870	6.1032E-02	3.0095E-02	1.4352E-02	2.8031E-03	4.1039E-04
.0960	1.556271	.950617	.558345	.251035	.124416	6.2674E-02	3.1110E-02	1.4943E-02	2.9672E-03	4.4315E-04
.0970	1.564412	.957506	.563909	.254825	.126969	6.4339E-02	3.2139E-02	1.5547E-02	3.1374E-03	4.7777E-04
.0980	1.572492	.964349	.569445	.258608	.129528	6.6011E-02	3.3182E-02	1.6162E-02	3.3136E-03	5.1431E-04
.0990	1.580514	.971149	.574954	.262385	.132093	6.7695E-02	3.4238E-02	1.6789E-02	3.4959E-03	5.5283E-04
.1000	1.588478	.977905	.580435	.266155	.134663	6.9390E-02	3.5306E-02	1.7428E-02	3.6845E-03	5.9339E-04
.1100	1.665165	1.043231	.633006	.303456	.160598	8.6902E-02	4.6657E-02	2.4432E-02	5.9177E-03	1.1232E-03
.1200	1.737082	1.104905	.684762	.339989	.186791	.105245	5.9065E-02	3.2473E-02	8.8009E-03	1.9143E-03
.1300	1.804937	1.163423	.733565	.375720	.213060	.124197	7.2338E-02	4.1427E-02	1.2338E-02	3.0102E-03
.1400	1.869291	1.219187	.780442	.410651	.239282	.143589	8.6317E-02	5.1175E-02	1.6514E-02	4.4431E-03
.1500	1.930593	1.272528	.825587	.444799	.265371	.163287	.100865	6.1608E-02	2.1299E-02	6.2348E-03
.1600	1.989215	1.323719	.869170	.478191	.291271	.183190	.115870	7.2629E-02	2.6657E-02	8.3974E-03
.1700	2.045460	1.372991	.911335	.510863	.316944	.203222	.131241	8.4151E-02	3.2549E-02	1.0935E-02
.1800	2.099584	1.420538	.952211	.542848	.342364	.223320	.146903	9.6101E-02	3.8932E-02	1.3844E-02
.1900	2.151805	1.466528	.991908	.574183	.367518	.243439	.162792	.108415	4.5766E-02	1.7119E-02
.2000	2.202305	1.511103	1.030526	.604902	.392398	.263544	.178859	.121038	5.3011E-02	2.0746E-02

TABLE A1 (CONTINUED)

a	y: 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.2100	2.251244	1.554388	1.068149	.635041	.417000	.283606	.195062	.133922	.060630	.024714
.2200	2.298758	1.596493	1.104856	.664630	.441326	.303605	.211366	.147027	.068589	.029007
.2300	2.344969	1.637511	1.140712	.693699	.465379	.323526	.227743	.160319	.076857	.033609
.2400	2.389980	1.677528	1.175781	.722278	.489164	.343355	.244169	.173768	.085405	.038503
.2500	2.433885	1.716618	1.210116	.750393	.512687	.363085	.260626	.187348	.094206	.043674
.2600	2.476767	1.754847	1.243767	.778068	.535954	.382709	.277098	.201039	.103236	.049105
.2700	2.518697	1.792275	1.276777	.805327	.558975	.402222	.293571	.214821	.112475	.054780
.2800	2.559743	1.828956	1.309187	.832190	.581755	.421622	.310034	.228678	.121902	.060686
.2900	2.599963	1.864938	1.341034	.858679	.604304	.440906	.326480	.242597	.131500	.066807
.3000	2.639409	1.900264	1.372350	.884811	.626628	.460074	.342899	.256565	.141253	.073131
.3100	2.678131	1.934974	1.403167	.910604	.648735	.479126	.359288	.270572	.151146	.079645
.3200	2.716171	1.969104	1.433511	.936074	.670632	.498062	.375640	.284609	.161168	.086336
.3300	2.753570	2.002688	1.463409	.961237	.692328	.516884	.391951	.298669	.171306	.093194
.3400	2.790363	2.035755	1.492884	.986106	.713829	.535593	.408220	.312745	.181549	.100208
.3500	2.826584	2.068333	1.521958	1.010695	.735141	.554190	.424442	.326831	.191888	.107369
.3600	2.862264	2.100447	1.550651	1.035016	.756272	.572677	.440616	.340922	.202315	.114668
.3700	2.897430	2.132122	1.578981	1.059081	.777228	.591057	.456740	.355013	.212822	.122096
.3800	2.932109	2.163379	1.606966	1.082900	.798014	.609331	.472814	.369102	.223401	.129645
.3900	2.966323	2.194237	1.634621	1.106484	.818637	.627502	.488836	.383184	.234047	.137308
.4000	3.000097	2.224716	1.661962	1.129843	.839102	.645571	.504806	.397257	.244752	.145078
.4100	3.033449	2.254833	1.689002	1.152985	.859414	.663541	.520724	.411319	.255513	.152949
.4200	3.066400	2.284604	1.715755	1.175918	.879579	.681415	.536589	.425367	.266324	.160915
.4300	3.098966	2.314045	1.742232	1.198652	.899601	.699194	.552402	.439400	.277181	.168970
.4400	3.131165	2.343168	1.768445	1.221193	.919485	.716881	.568163	.453416	.288079	.177110
.4500	3.163012	2.371988	1.794404	1.243549	.939235	.734478	.583871	.467414	.299016	.185328
.4600	3.194522	2.400517	1.820120	1.265726	.958856	.751987	.599529	.481393	.309987	.193622
.4700	3.225708	2.428767	1.845602	1.287731	.978352	.769411	.615135	.495351	.320990	.201987
.4800	3.256584	2.456747	1.870858	1.309570	.997726	.786751	.630690	.509288	.332022	.210419
.4900	3.287160	2.484470	1.895898	1.331248	1.016983	.804009	.646196	.523204	.343081	.218913
.5000	3.317448	2.511943	1.920729	1.352772	1.036125	.821187	.661652	.537097	.354163	.227468
.5100	3.347460	2.539177	1.945359	1.374145	1.055157	.838288	.677059	.550968	.365267	.236080
.5200	3.377204	2.566180	1.969794	1.395374	1.074082	.855313	.692419	.564815	.376391	.244745
.5300	3.406691	2.592959	1.994042	1.416462	1.092902	.872264	.707731	.578640	.387534	.253461
.5400	3.435930	2.619524	2.018108	1.437415	1.111621	.889143	.722997	.592441	.398692	.262226
.5500	3.464928	2.645880	2.041999	1.458236	1.130241	.905951	.738216	.606218	.409866	.271036
.5600	3.493695	2.672036	2.065720	1.478930	1.148766	.922690	.753391	.619971	.421052	.279891
.5700	3.522238	2.697997	2.089276	1.499500	1.167198	.939363	.768521	.633701	.432251	.288786
.5800	3.550563	2.723769	2.112674	1.519950	1.185539	.955969	.783608	.647407	.443461	.297721
.5900	3.578678	2.749360	2.135918	1.540283	1.203793	.972512	.798652	.661089	.454681	.306694
.6000	3.606589	2.774773	2.159012	1.560503	1.221960	.988992	.813654	.674748	.465909	.315702
.6100	3.634303	2.800016	2.181961	1.580614	1.240044	1.005411	.828614	.688383	.477146	.324744
.6200	3.661826	2.825091	2.204769	1.600617	1.258047	1.021770	.843533	.701995	.488389	.333819
.6300	3.689162	2.850006	2.227441	1.620517	1.275970	1.038070	.858413	.715583	.499638	.342925
.6400	3.716318	2.874764	2.249980	1.640316	1.293817	1.054314	.873253	.729148	.510893	.352060
.6500	3.743299	2.899370	2.272389	1.660016	1.311588	1.070502	.888055	.742690	.522152	.361224
.6600	3.770109	2.923827	2.294673	1.679620	1.329285	1.086635	.902818	.756210	.533416	.370415
.6700	3.796754	2.948141	2.316835	1.699131	1.346911	1.102714	.917545	.769706	.544683	.379631
.6800	3.823238	2.972314	2.338878	1.718552	1.364467	1.118742	.932235	.783181	.555953	.388872
.6900	3.849565	2.996352	2.360806	1.737883	1.381955	1.134718	.946889	.796633	.567226	.398137
.7000	3.875739	3.020256	2.382620	1.757129	1.399376	1.150644	.961507	.810064	.578500	.407424

TABLE A1 (CONTINUED)

a	γ = 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
.7100	3.901764	3.044032	2.404325	1.776289	1.416732	1.166522	.976091	.823473	.589776	.416733
.7200	3.927444	3.067681	2.425922	1.795368	1.434024	1.182351	.990641	.836860	.601053	.426062
.7300	3.953383	3.091207	2.447415	1.814366	1.451254	1.198133	1.005157	.850226	.612331	.435412
.7400	3.978985	3.114614	2.468806	1.833286	1.468424	1.213869	1.019640	.863571	.623609	.444780
.7500	4.004452	3.137904	2.490098	1.852130	1.485533	1.229561	1.034091	.876895	.634888	.454167
.7600	4.029787	3.161080	2.511292	1.870899	1.502585	1.245208	1.048511	.890199	.646166	.463571
.7700	4.054995	3.184144	2.532392	1.889594	1.519580	1.260811	1.062899	.903483	.657444	.472993
.7800	4.080078	3.207100	2.553399	1.908218	1.536519	1.276373	1.077256	.916747	.668721	.482430
.7900	4.105038	3.229949	2.574315	1.926773	1.553403	1.291892	1.091583	.929991	.679996	.491883
.8000	4.129879	3.252695	2.595144	1.945250	1.570234	1.307371	1.105881	.943215	.691271	.501351
.8100	4.154603	3.275339	2.615885	1.963677	1.587013	1.322810	1.120149	.956420	.702545	.510834
.8200	4.179214	3.297884	2.636542	1.982031	1.603741	1.338209	1.134388	.969607	.713816	.520330
.8300	4.203712	3.320331	2.657117	2.000321	1.620418	1.353570	1.148600	.982774	.725086	.529840
.8400	4.228102	3.342684	2.677610	2.018547	1.637046	1.368894	1.162784	.995923	.736355	.539362
.8500	4.252384	3.364944	2.698024	2.036713	1.653626	1.384180	1.176940	1.009054	.747621	.548897
.8600	4.276562	3.387113	2.718360	2.054818	1.670159	1.399429	1.191070	1.022166	.758884	.558444
.8700	4.300638	3.409192	2.738621	2.072864	1.686646	1.414643	1.205173	1.035261	.770146	.568003
.8800	4.324613	3.431185	2.758807	2.090852	1.703086	1.429822	1.219250	1.048338	.781405	.577572
.8900	4.348490	3.453092	2.778920	2.108784	1.719483	1.444966	1.233302	1.061398	.792661	.587152
.9000	4.372270	3.474915	2.798961	2.126660	1.735835	1.460076	1.247328	1.074441	.803915	.596743
.9100	4.395956	3.496656	2.818932	2.144482	1.752145	1.475153	1.261330	1.087467	.815166	.606344
.9200	4.419550	3.518317	2.838835	2.162250	1.768412	1.490198	1.275308	1.100476	.826415	.615954
.9300	4.443052	3.539898	2.858670	2.179966	1.784638	1.505210	1.289261	1.113468	.837660	.625574
.9400	4.466466	3.561402	2.878439	2.197630	1.800823	1.520190	1.303191	1.126445	.848902	.635202
.9500	4.489793	3.582830	2.898144	2.215243	1.816969	1.535140	1.317098	1.139405	.860142	.644839
.9600	4.513034	3.604184	2.917784	2.232808	1.833075	1.550058	1.330981	1.152350	.871378	.654485
.9700	4.536191	3.625464	2.937362	2.250323	1.849142	1.564947	1.344843	1.165278	.882611	.664139
.9800	4.559264	3.646673	2.956879	2.267790	1.865172	1.579806	1.358682	1.178192	.893841	.673801
.9900	4.582257	3.667811	2.976335	2.285211	1.881154	1.594636	1.372499	1.191090	.905067	.683470
1.0000	4.605170	3.688879	2.995732	2.302585	1.897120	1.609438	1.386294	1.203973	.916291	.693147
1.1000	4.830207	3.896009	3.186665	2.473953	2.054791	1.755974	1.523133	1.332016	1.028337	.790275
1.2000	5.048610	4.097368	3.372663	2.641460	2.209386	1.900089	1.658130	1.458746	1.140034	.887936
1.3000	5.261312	4.293777	3.554429	2.805648	2.361325	2.042103	1.791513	1.584312	1.251383	.985999
1.4000	5.469054	4.485882	3.732515	2.966944	2.510947	2.182274	1.923476	1.708839	1.362394	1.084372
1.5000	5.672434	4.674202	3.907365	3.125694	2.658524	2.320814	2.054173	1.832435	1.473083	1.182987
1.6000	5.871938	4.859161	4.079340	3.282184	2.804280	2.457898	2.183735	1.955190	1.583468	1.281796
1.7000	6.067967	5.041111	4.248742	3.436646	2.948404	2.593675	2.312273	2.077183	1.693566	1.380762
1.8000	6.260862	5.220348	4.415828	3.589282	3.091050	2.728265	2.439884	2.198480	1.803395	1.479857
1.9000	6.450908	5.397120	4.580807	3.740258	3.232358	2.861780	2.566647	2.319141	1.912973	1.579058
2.0000	6.638354	5.571643	4.743866	3.889720	3.372440	2.994308	2.692635	2.439216	2.022313	1.678347
2.1000	6.823411	5.744103	4.905160	4.037792	3.511401	3.125932	2.817907	2.558751	2.131432	1.777712
2.2000	7.006262	5.914661	5.064831	4.184584	3.649324	3.256717	2.942518	2.677785	2.240341	1.877141
2.3000	7.187075	6.083457	5.223002	4.330193	3.786287	3.386729	3.066517	2.796353	2.349054	1.976625
2.4000	7.365991	6.250616	5.379776	4.474698	3.922359	3.516020	3.189945	2.914488	2.457581	2.076157
2.5000	7.543136	6.416252	5.535249	4.618178	4.057600	3.644637	3.312840	3.032216	2.565934	2.175730
2.6000	7.718626	6.580461	5.689507	4.760700	4.192063	3.772629	3.435236	3.149562	2.674120	2.275339
2.7000	7.892568	6.743333	5.842625	4.902326	4.325799	3.900030	3.557164	3.266552	2.782151	2.374980
2.8000	8.065047	6.904947	5.994674	5.043103	4.458850	4.026877	3.678652	3.383203	2.890033	2.474650
2.9000	8.236147	7.065379	6.145709	5.183088	4.591255	4.153199	3.799724	3.499536	2.997774	2.574344
3.0000	8.405951	7.224687	6.295794	5.322320	4.723050	4.279030	3.920403	3.615568	3.105379	2.674061

TABLE A1 (CONTINUED)

a	γ 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
3.1000	8.574519	7.382939	6.444977	5.460843	4.854271	4.404393	4.040710	3.731314	3.212857	2.773797
3.2000	8.741913	7.540188	6.593303	5.598691	4.984944	4.529312	4.160664	3.846789	3.320213	2.873551
3.3000	8.908206	7.696479	6.740820	5.735901	5.115098	4.653811	4.280279	3.962007	3.427453	2.973322
3.4000	9.073433	7.851864	6.887563	5.872497	5.244759	4.777909	4.399579	4.076978	3.534582	3.073107
3.5000	9.237654	8.006381	7.033568	6.008518	5.373948	4.901625	4.518574	4.191715	3.641604	3.172907
3.6000	9.400913	8.160075	7.178876	6.143985	5.502689	5.024977	4.637277	4.306229	3.748524	3.272717
3.7000	9.563252	8.312978	7.323508	6.278921	5.631001	5.147980	4.755703	4.420530	3.855346	3.372538
3.8000	9.724709	8.465125	7.467498	6.413355	5.758900	5.270650	4.873863	4.534621	3.962074	3.472370
3.9000	9.885325	8.616545	7.610876	6.547301	5.886407	5.393001	4.991767	4.648519	4.068712	3.572211
4.0000	10.045118	8.767274	7.753657	6.680783	6.013537	5.515046	5.109430	4.762229	4.175263	3.672061
4.1000	10.204140	8.917333	7.895874	6.813819	6.140305	5.636796	5.226851	4.875757	4.281728	3.771918
4.2000	10.362412	9.066751	8.037546	6.946426	6.266724	5.758266	5.344051	4.989110	4.388118	3.871783
4.3000	10.519965	9.215550	8.178691	7.078618	6.392811	5.879458	5.461033	5.102295	4.494426	3.971656
4.4000	10.676814	9.363754	8.319335	7.210415	6.518569	6.000392	5.577805	5.215319	4.600659	4.071533
4.5000	10.832998	9.511386	8.459489	7.341828	6.644020	6.121073	5.694376	5.328186	4.706820	4.171417
4.6000	10.988535	9.658461	8.599174	7.472874	6.769170	6.241509	5.810751	5.440905	4.812911	4.271305
4.7000	11.143445	9.805005	8.738404	7.603558	6.894031	6.361711	5.926936	5.553474	4.918937	4.371201
4.8000	11.297752	9.951026	8.877196	7.733902	7.018612	6.481687	6.042944	5.665904	5.024892	4.471100
4.9000	11.451474	10.096550	9.015564	7.863905	7.142921	6.601438	6.158772	5.778199	5.130786	4.571002
5.0000	11.604626	10.241591	9.153519	7.993589	7.266964	6.720979	6.274431	5.890363	5.236618	4.670909
5.5000	12.362488	10.960025	9.837569	8.637505	7.883548	7.315710	6.850347	6.449336	5.764916	5.170499
6.0000	13.108487	11.668333	10.513032	9.274674	8.494657	7.905993	7.422702	7.005551	6.291919	5.670162
6.5000	13.844121	12.367804	11.181017	9.905965	9.100988	8.492397	7.991953	7.559361	6.817786	6.169878
7.0000	14.570621	13.059480	11.842396	10.532072	9.703121	9.075384	8.558467	8.111049	7.342646	6.669638
7.5000	15.288963	13.744197	12.497896	11.153566	10.301504	9.655329	9.122543	8.660847	7.866611	7.169430
8.0000	15.999966	14.422678	13.148115	11.770915	10.894529	10.232544	9.684431	9.208947	8.389768	7.669251
8.5000	16.704332	15.095506	13.793557	12.384520	11.488514	10.807280	10.244338	9.755507	8.912195	8.169092
9.0000	17.402656	15.763192	14.434646	12.994712	12.077736	11.379774	10.802445	10.300677	9.433952	8.668950
9.5000	18.095438	16.426164	15.071766	13.601792	12.664427	11.950209	11.358903	10.844563	9.955096	9.168827
10.0000	18.783123	17.084810	15.705213	14.205991	13.248793	12.518752	11.913846	11.387274	10.475684	9.668716
11.0000	20.144684	18.390357	16.962222	15.406642	14.411228	13.650727	13.019633	12.469508	11.515331	10.668524
12.0000	21.489914	19.682034	18.207521	16.598123	15.566237	14.776661	14.120579	13.547985	12.553176	11.668363
13.0000	22.820845	20.961583	19.442570	17.781586	16.714734	15.897311	15.217283	14.623163	13.589441	12.668229
14.0000	24.139129	22.230398	20.668571	18.957964	17.857494	17.013283	16.310247	15.695437	14.624309	13.668115
15.0000	25.446105	23.489624	21.886488	20.128012	18.995128	18.125096	17.399879	16.765117	15.657928	14.668016
16.0000	26.742891	24.740213	23.097125	21.292374	20.128149	19.233157	18.486495	17.832455	16.690432	15.667930
17.0000	28.030459	25.982992	24.301184	22.451580	21.257004	20.337825	19.570390	18.897693	17.721915	16.667854
18.0000	29.309600	27.218645	25.499231	23.606091	22.382035	21.439396	20.651808	19.960991	18.752470	17.667791
19.0000	30.581062	28.447777	26.691771	24.756292	[illegible]3.503585	22.538139	21.730954	21.022525	19.782180	18.667726
20.0000	31.845381	29.670858	27.879242	25.902540	24.621929	23.634281	22.808003	22.082438	20.811097	19.667673
21.0000	33.103128	30.888383	29.062021	27.045102	25.737296	24.727986	23.883126	23.140838	21.839302	20.667624
22.0000	34.354764	32.100744	30.240440	28.184271	26.849910	25.819462	24.956450	24.197847	22.866819	21.667583
23.0000	35.600707	33.308266	31.414812	29.320269	27.959956	26.908856	26.028098	25.253553	23.893720	22.667548
24.0000	36.841330	34.511301	32.585374	30.453305	29.067603	27.996296	27.098184	26.308045	24.920029	23.667503
25.0000	38.076956	35.710101	33.752414	31.583561	30.172998	29.081890	28.166803	27.361398	25.945792	24.667469
26.0000	39.307887	36.904948	34.916089	32.711215	31.276287	30.165788	29.234045	28.413679	26.971026	25.667437
27.0000	40.534390	38.096045	36.076610	33.836394	32.377578	31.248064	30.299991	29.464952	27.995799	26.667405
28.0000	41.756738	39.283586	37.234173	34.959258	33.476977	32.328819	31.364711	30.515272	29.020101	27.667379
29.0000	42.975095	40.467809	38.388918	36.079917	34.574607	33.408109	32.428274	31.564690	30.043968	28.667350
30.0000	44.189717	41.648842	39.540987	37.198505	35.670551	34.486044	33.490732	32.613239	31.067421	29.667329

TABLE A1 (CONTINUED)

α	γ 0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
31.0000	45.400776	42.826876	40.690505	38.315117	36.764887	35.562657	34.552154	33.661009	32.090473	30.667311
32.0000	46.608444	44.002030	41.837637	39.429823	37.857699	36.638052	35.612575	34.707970	33.113172	31.667291
33.0000	47.812869	45.174458	42.982466	40.542744	38.949060	37.712247	36.672057	35.754229	34.135515	32.667272
34.0000	49.014226	46.344288	44.125085	41.653939	40.039020	38.785329	37.730619	36.799770	35.157496	33.667258
35.0000	50.212588	47.511599	45.265616	42.763521	41.127677	39.857326	38.788329	37.844639	36.179174	34.667230
36.0000	51.408187	48.676511	46.404120	43.871508	42.215054	40.928296	39.845226	38.888883	37.200535	35.667222
37.0000	52.601029	49.839183	47.540735	44.978029	43.301215	41.998286	40.901293	39.932480	38.221599	36.667205
38.0000	53.791297	50.999632	48.675488	46.083084	44.386209	43.067307	41.956629	40.975497	39.242375	37.667198
39.0000	54.979045	52.157977	49.808471	47.186744	45.470082	44.135427	43.011231	42.017968	40.262888	38.667178
40.0000	56.164393	53.314291	50.939746	48.289090	46.552879	45.202675	44.065116	43.059856	41.283126	39.667166
41.0000	57.347484	54.468656	52.069372	49.390155	47.634639	46.269059	45.118356	44.101228	42.303114	40.667153
42.0000	58.528289	55.621137	53.197424	50.490013	48.715385	47.334670	46.170926	45.142096	43.322840	41.667142
43.0000	59.706962	56.771796	54.323951	51.588637	49.795186	48.399482	47.222881	46.182505	44.342367	42.667131
44.0000	60.883584	57.920722	55.449005	52.686107	50.874063	49.463514	48.274212	47.222420	45.361649	43.667121
45.0000	62.058173	59.067953	56.572638	53.782505	51.952057	50.526864	49.324967	48.261888	46.380712	44.667110
46.0000	63.230868	60.213551	57.694899	54.877815	53.029132	51.589485	50.375165	49.300905	47.399561	45.667100
47.0000	64.401640	61.357533	58.815828	55.972092	54.105397	52.651420	51.424800	50.339508	48.418206	46.667091
48.0000	65.570640	62.500050	59.935454	57.065355	55.180848	53.712694	52.473897	51.377691	49.436650	47.667082
49.0000	66.737841	63.641014	61.053858	58.157640	56.255501	54.773339	53.522507	52.415459	50.454918	48.667073
50.0000	67.903375	64.780609	62.171059	59.249005	57.329410	55.833358	54.570622	53.452881	51.472973	49.667065

TABLE A2. **Gamma Reference Scale Values b_0 for Selected Values of Shape Parameter (a) and Prior Assurance (p_0)**

a/p_0	0.95	0.90	0.80
0.01	3.5227-155	4.4655-125	5.6607E-95
0.02	4.5019E-75	5.0687E-60	5.7068E-45
0.03	2.2797E-48	2.4673E-38	2.6702E-28
0.04	5.1503E-35	1.7282E-27	5.7988E-20
0.05	5.3157E-27	5.5739E-21	5.8446E-15
0.06	1.1726E-21	1.2199E-16	1.2691E-11
0.07	7.7079E-18	1.5394E-13	3.0746E-09
0.08	5.6375E-15	3.2656E-11	1.8916E-07
0.09	9.5383E-13	2.1098E-09	4.6669E-06
0.10	5.7917E-11	5.9307E-08	6.0730E-05
0.11	1.6690E-09	9.1013E-07	4.9629E-04
0.12	2.7502E-08	8.8704E-06	2.8611E-03
0.13	2.9479E-07	6.0971E-05	1.2611E-02
0.14	2.2538E-06	3.1852E-04	4.5014E-02
0.15	1.3150E-05	1.3360E-03	1.3573E-01
0.16	6.1598E-05	4.6882E-03	3.5682E-01
0.17	2.4079E-04	1.4204E-02	8.3786E-01
0.18	8.0955E-04	3.8074E-02	1.7907E+00
0.19	2.3972E-03	9.2062E-02	3.5355E+00
0.20	6.3725E-03	2.0392E-01	6.5255E+00
0.21	1.5443E-02	4.1900E-01	1.1368E+01
0.22	3.4551E-02	8.0682E-01	1.8841E+01
0.23	7.2112E-02	1.4684E+00	2.9900E+01
0.24	1.4163E-01	2.5435E+00	4.5682E+01
0.25	2.6366E-01	4.2186E+00	6.7501E+01
0.26	4.6815E-01	6.7327E+00	9.6835E+01
0.27	7.9697E-01	1.0384E+01	1.3531E+02
0.28	1.3067E+00	1.5534E+01	1.8469E+02
0.29	2.0715E+00	2.2611E+01	2.4685E+02
0.30	3.1857E+00	3.2110E+01	3.2372E+02
0.31	4.7668E+00	4.4597E+01	4.1734E+02
0.32	6.9577E+00	6.0702E+01	5.2976E+02
0.33	9.9288E+00	8.1121E+01	6.6303E+02
0.34	1.3880E+01	1.0661E+02	8.1924E+02
0.35	1.9041E+01	1.3798E+02	1.0004E+03
0.36	2.5675E+01	1.7610E+02	1.2086E+03
0.37	3.4074E+01	2.2186E+02	1.4457E+03
0.38	4.4566E+01	2.7622E+02	1.7135E+03
0.39	5.7506E+01	3.4015E+02	2.0140E+03
0.40	7.3283E+01	4.1465E+02	2.3489E+03
0.41	9.2314E+01	5.0074E+02	2.7197E+03
0.42	1.1504E+02	5.9945E+02	3.1280E+03
0.43	1.4194E+02	7.1180E+02	3.5752E+03
0.44	1.7351E+02	8.3884E+02	4.0627E+03
0.45	2.1026E+02	9.8160E+02	4.5917E+03
0.46	2.5272E+02	1.1411E+03	5.1633E+03
0.47	3.0146E+02	1.3183E+03	5.7786E+03
0.48	3.5703E+02	1.5142E+03	6.4385E+03
0.49	4.2003E+02	1.7298E+03	7.1438E+03
0.50	4.9103E+02	1.9661E+03	7.8954E+03
0.51	5.7065E+02	2.2238E+03	8.6938E+03
0.52	6.5947E+02	2.5039E+03	9.5397E+03
0.53	7.5810E+02	2.8073E+03	1.0434E+04
0.54	8.6715E+02	3.1347E+03	1.1376E+04
0.55	9.8723E+02	3.4869E+03	1.2367E+04
0.56	1.1189E+03	3.8647E+03	1.3407E+04
0.57	1.2628E+03	4.2688E+03	1.4496E+04
0.58	1.4196E+03	4.6997E+03	1.5635E+04
0.59	1.5897E+03	5.1582E+03	1.6823E+04
0.60	1.7737E+03	5.6448E+03	1.8060E+04

Table A2 (Continued)

a/p_0	0.95	0.90	0.80
0.61	1.9723E+03	6.1602E+03	1.9348E+04
0.62	2.1859E+03	6.7047E+03	2.0685E+04
0.63	2.4152E+03	7.2789E+03	2.2071E+04
0.64	2.6605E+03	7.8833E+03	2.3506E+04
0.65	2.9225E+03	8.5182E+03	2.4991E+04
0.66	3.2016E+03	9.1840E+03	2.6525E+04
0.67	3.4982E+03	9.8811E+03	2.8108E+04
0.68	3.8130E+03	1.0610E+04	2.9739E+04
0.69	4.1462E+03	1.1370E+04	3.1418E+04
0.70	4.4983E+03	1.2163E+04	3.3145E+04
0.71	4.8698E+03	1.2988E+04	3.4921E+04
0.72	5.2610E+03	1.3846E+04	3.6743E+04
0.73	5.6722E+03	1.4737E+04	3.8612E+04
0.74	6.1040E+03	1.5660E+04	4.0528E+04
0.75	6.5565E+03	1.6616E+04	4.2491E+04
0.76	7.0302E+03	1.7606E+04	4.4499E+04
0.77	7.5253E+03	1.8629E+04	4.6552E+04
0.78	8.0421E+03	1.9685E+04	4.8651E+04
0.79	8.5809E+03	2.0775E+04	5.0794E+04
0.80	9.1420E+03	2.1898E+04	5.2982E+04
0.81	9.7256E+03	2.3054E+04	5.5213E+04
0.82	1.0332E+04	2.4244E+04	5.7488E+04
0.83	1.0961E+04	2.5467E+04	5.9806E+04
0.84	1.1614E+04	2.6723E+04	6.2166E+04
0.85	1.2289E+04	2.8013E+04	6.4569E+04
0.86	1.2989E+04	2.9336E+04	6.7013E+04
0.87	1.3712E+04	3.0692E+04	6.9498E+04
0.88	1.4459E+04	3.2082E+04	7.2024E+04
0.89	1.5229E+04	3.3504E+04	7.4590E+04
0.90	1.6024E+04	3.4959E+04	7.7197E+04
0.91	1.6843E+04	3.6448E+04	7.9842E+04
0.92	1.7687E+04	3.7968E+04	8.2527E+04
0.93	1.8555E+04	3.9522E+04	8.5251E+04
0.94	1.9447E+04	4.1107E+04	8.8012E+04
0.95	2.0364E+04	4.2725E+04	9.0812E+04
0.96	2.1305E+04	4.4375E+04	9.3649E+04
0.97	2.2271E+04	4.6057E+04	9.6522E+04
0.98	2.3262E+04	4.7771E+04	9.9432E+04
0.99	2.4278E+04	4.9516E+04	1.0238E+05
1.00	2.5318E+04	5.1293E+04	1.0536E+05
1.05	3.0891E+04	6.0643E+04	1.2079E+05
1.10	3.7082E+04	7.0752E+04	1.3705E+05
1.15	4.3888E+04	8.1594E+04	1.5410E+05
1.20	5.1301E+04	9.3145E+04	1.7190E+05
1.25	5.9311E+04	1.0538E+05	1.9039E+05
1.30	6.7908E+04	1.1827E+05	2.0956E+05
1.35	7.7079E+04	1.3179E+05	2.2935E+05
1.40	8.6810E+04	1.4593E+05	2.4974E+05
1.45	9.7087E+04	1.6064E+05	2.7069E+05
1.50	1.0790E+05	1.7592E+05	2.9219E+05
1.55	1.1923E+05	1.9174E+05	3.1419E+05
1.60	1.3106E+05	2.0808E+05	3.3669E+05
1.65	1.4338E+05	2.2492E+05	3.5965E+05
1.70	1.5618E+05	2.4225E+05	3.8306E+05
1.75	1.6944E+05	2.6003E+05	4.0689E+05
1.80	1.8314E+05	2.7827E+05	4.3113E+05
1.85	1.9729E+05	2.9693E+05	4.5575E+05
1.90	2.1186E+05	3.1601E+05	4.8075E+05
1.95	2.2683E+05	3.3549E+05	5.0611E+05
2.00	2.4221E+05	3.5536E+05	5.3181E+05

Table A2 (Continued)

a/p_0	0.95	0.90	0.80
2.05	2.5797E+05	3.7560E+05	5.5784E+05
2.10	2.7411E+05	3.9621E+05	5.8419E+05
2.15	2.9062E+05	4.1716E+05	6.1085E+05
2.20	3.0748E+05	4.3845E+05	6.3780E+05
2.25	3.2469E+05	4.6007E+05	6.6504E+05
2.30	3.4224E+05	4.8201E+05	6.9255E+05
2.35	3.6011E+05	5.0425E+05	7.2032E+05
2.40	3.7830E+05	5.2679E+05	7.4835E+05
2.45	3.9680E+05	5.4963E+05	7.7663E+05
2.50	4.1561E+05	5.7274E+05	8.0515E+05
2.55	4.3470E+05	5.9612E+05	8.3391E+05
2.60	4.5409E+05	6.1977E+05	8.6288E+05
2.65	4.7375E+05	6.4368E+05	8.9208E+05
2.70	4.9369E+05	6.6784E+05	9.2149E+05
2.75	5.1389E+05	6.9224E+05	9.5110E+05
2.80	5.3435E+05	7.1688E+05	9.8092E+05
2.85	5.5507E+05	7.4175E+05	1.0109E+06
2.90	5.7603E+05	7.6685E+05	1.0411E+06
2.95	5.9723E+05	7.9216E+05	1.0715E+06
3.00	6.1867E+05	8.1769E+05	1.1021E+06
3.05	6.4034E+05	8.4343E+05	1.1328E+06
3.10	6.6224E+05	8.6937E+05	1.1637E+06
3.15	6.8435E+05	8.9551E+05	1.1948E+06
3.20	7.0668E+05	9.2184E+05	1.2260E+06
3.25	7.2922E+05	9.4837E+05	1.2574E+06
3.30	7.5197E+05	9.7507E+05	1.2889E+06
3.35	7.7492E+05	1.0020E+06	1.3206E+06
3.40	7.9807E+05	1.0290E+06	1.3525E+06
3.45	8.2141E+05	1.0563E+06	1.3844E+06
3.50	8.4493E+05	1.0837E+06	1.4166E+06
3.55	8.6865E+05	1.1112E+06	1.4488E+06
3.60	8.9255E+05	1.1390E+06	1.4812E+06
3.65	9.1662E+05	1.1669E+06	1.5137E+06
3.70	9.4087E+05	1.1949E+06	1.5464E+06
3.75	9.6529E+05	1.2231E+06	1.5791E+06
3.80	9.8988E+05	1.2515E+06	1.6120E+06
3.85	1.0146E+06	1.2800E+06	1.6450E+06
3.90	1.0396E+06	1.3086E+06	1.6782E+06
3.95	1.0646E+06	1.3374E+06	1.7114E+06
4.00	1.0899E+06	1.3663E+06	1.7448E+06
4.05	1.1153E+06	1.3954E+06	1.7782E+06
4.10	1.1408E+06	1.4246E+06	1.8118E+06
4.15	1.1665E+06	1.4539E+06	1.8455E+06
4.20	1.1923E+06	1.4833E+06	1.8793E+06
4.25	1.2183E+06	1.5129E+06	1.9132E+06
4.30	1.2444E+06	1.5426E+06	1.9472E+06
4.35	1.2706E+06	1.5724E+06	1.9812E+06
4.40	1.2970E+06	1.6023E+06	2.0154E+06
4.45	1.3235E+06	1.6324E+06	2.0497E+06
4.50	1.3502E+06	1.6626E+06	2.0841E+06
4.55	1.3770E+06	1.6928E+06	2.1185E+06
4.60	1.4039E+06	1.7232E+06	2.1531E+06
4.65	1.4309E+06	1.7537E+06	2.1877E+06
4.70	1.4581E+06	1.7843E+06	2.2225E+06
4.75	1.4853E+06	1.8150E+06	2.2573E+06
4.80	1.5127E+06	1.8459E+06	2.2922E+06
4.85	1.5403E+06	1.8768E+06	2.3272E+06
4.90	1.5679E+06	1.9078E+06	2.3622E+06
4.95	1.5956E+06	1.9389E+06	2.3974E+06
5.00	1.6235E+06	1.9701E+06	2.4326E+06

Table A2 (Continued)

a/p_0	0.95	0.90	0.80
5.05	1.6515E+06	2.0015E+06	2.4679E+06
5.10	1.6795E+06	2.0329E+06	2.5033E+06
5.15	1.7077E+06	2.0644E+06	2.5387E+06
5.20	1.7360E+06	2.0960E+06	2.5742E+06
5.25	1.7644E+06	2.1277E+06	2.6098E+06
5.30	1.7929E+06	2.1594E+06	2.6455E+06
5.35	1.8215E+06	2.1913E+06	2.6813E+06
5.40	1.8502E+06	2.2232E+06	2.7171E+06
5.45	1.8790E+06	2.2553E+06	2.7529E+06
5.50	1.9079E+06	2.2874E+06	2.7889E+06
5.55	1.9369E+06	2.3196E+06	2.8249E+06
5.60	1.9659E+06	2.3519E+06	2.8610E+06
5.65	1.9951E+06	2.3843E+06	2.8971E+06
5.70	2.0244E+06	2.4167E+06	2.9333E+06
5.75	2.0538E+06	2.4492E+06	2.9696E+06
5.80	2.0832E+06	2.4818E+06	3.0060E+06
5.85	2.1128E+06	2.5145E+06	3.0423E+06
5.90	2.1424E+06	2.5473E+06	3.0788E+06
5.95	2.1721E+06	2.5801E+06	3.1153E+06
6.00	2.2019E+06	2.6130E+06	3.1519E+06
6.05	2.2318E+06	2.6460E+06	3.1885E+06
6.10	2.2617E+06	2.6790E+06	3.2252E+06
6.15	2.2918E+06	2.7122E+06	3.2620E+06
6.20	2.3219E+06	2.7454E+06	3.2988E+06
6.25	2.3521E+06	2.7786E+06	3.3356E+06
6.30	2.3824E+06	2.8119E+06	3.3726E+06
6.35	2.4128E+06	2.8453E+06	3.4095E+06
6.40	2.4433E+06	2.8788E+06	3.4465E+06
6.45	2.4738E+06	2.9123E+06	3.4836E+06
6.50	2.5044E+06	2.9459E+06	3.5208E+06
6.55	2.5351E+06	2.9796E+06	3.5579E+06
6.60	2.5658E+06	3.0133E+06	3.5952E+06
6.65	2.5966E+06	3.0471E+06	3.6324E+06
6.70	2.6275E+06	3.0810E+06	3.6698E+06
6.75	2.6585E+06	3.1149E+06	3.7072E+06
6.80	2.6895E+06	3.1488E+06	3.7446E+06
6.85	2.7206E+06	3.1829E+06	3.7821E+06
6.90	2.7518E+06	3.2170E+06	3.8196E+06
6.95	2.7831E+06	3.2511E+06	3.8572E+06
7.00	2.8144E+06	3.2853E+06	3.8948E+06
7.05	2.8457E+06	3.3196E+06	3.9324E+06
7.10	2.8772E+06	3.3539E+06	3.9701E+06
7.15	2.9087E+06	3.3883E+06	4.0079E+06
7.20	2.9403E+06	3.4227E+06	4.0457E+06
7.25	2.9719E+06	3.4572E+06	4.0835E+06
7.30	3.0036E+06	3.4918E+06	4.1214E+06
7.35	3.0354E+06	3.5264E+06	4.1593E+06
7.40	3.0672E+06	3.5610E+06	4.1973E+06
7.45	3.0991E+06	3.5957E+06	4.2353E+06
7.50	3.1311E+06	3.6305E+06	4.2734E+06
7.55	3.1631E+06	3.6653E+06	4.3115E+06
7.60	3.1952E+06	3.7001E+06	4.3496E+06
7.65	3.2273E+06	3.7351E+06	4.3878E+06
7.70	3.2595E+06	3.7700E+06	4.4260E+06
7.75	3.2917E+06	3.8050E+06	4.4643E+06
7.80	3.3240E+06	3.8401E+06	4.5026E+06
7.85	3.3564E+06	3.8752E+06	4.5409E+06
7.90	3.3888E+06	3.9104E+06	4.5793E+06
7.95	3.4213E+06	3.9456E+06	4.6177E+06
8.00	3.4538E+06	3.9808E+06	4.6561E+06

Table A2 (Continued)

a/p_0	0.95	0.90	0.80
8.05	3.4864E+06	4.0161E+06	4.6946E+06
8.10	3.5191E+06	4.0515E+06	4.7331E+06
8.15	3.5518E+06	4.0869E+06	4.7717E+06
8.20	3.5845E+06	4.1223E+06	4.8103E+06
8.25	3.6173E+06	4.1578E+06	4.8489E+06
8.30	3.6502E+06	4.1933E+06	4.8876E+06
8.35	3.6831E+06	4.2289E+06	4.9263E+06
8.40	3.7160E+06	4.2645E+06	4.9650E+06
8.45	3.7490E+06	4.3002E+06	5.0038E+06
8.50	3.7821E+06	4.3359E+06	5.0426E+06
8.55	3.8152E+06	4.3716E+06	5.0814E+06
8.60	3.8484E+06	4.4074E+06	5.1203E+06
8.65	3.8816E+06	4.4433E+06	5.1592E+06
8.70	3.9148E+06	4.4791E+06	5.1982E+06
8.75	3.9481E+06	4.5150E+06	5.2371E+06
8.80	3.9815E+06	4.5510E+06	5.2761E+06
8.85	4.0149E+06	4.5870E+06	5.3152E+06
8.90	4.0483E+06	4.6230E+06	5.3542E+06
8.95	4.0818E+06	4.6591E+06	5.3933E+06
9.00	4.1154E+06	4.6952E+06	5.4325E+06
9.05	4.1490E+06	4.7314E+06	5.4716E+06
9.10	4.1826E+06	4.7676E+06	5.5108E+06
9.15	4.2163E+06	4.8038E+06	5.5500E+06
9.20	4.2500E+06	4.8401E+06	5.5893E+06
9.25	4.2838E+06	4.8764E+06	5.6286E+06
9.30	4.3176E+06	4.9127E+06	5.6679E+06
9.35	4.3514E+06	4.9491E+06	5.7072E+06
9.40	4.3853E+06	4.9856E+06	5.7466E+06
9.45	4.4193E+06	5.0220E+06	5.7860E+06
9.50	4.4533E+06	5.0585E+06	5.8255E+06
9.55	4.4873E+06	5.0950E+06	5.8649E+06
9.60	4.5214E+06	5.1316E+06	5.9044E+06
9.65	4.5555E+06	5.1682E+06	5.9439E+06
9.70	4.5896E+06	5.2048E+06	5.9835E+06
9.75	4.6238E+06	5.2415E+06	6.0230E+06
9.80	4.6580E+06	5.2782E+06	6.0626E+06
9.85	4.6923E+06	5.3150E+06	6.1023E+06
9.90	4.7266E+06	5.3518E+06	6.1419E+06
9.95	4.7610E+06	5.3886E+06	6.1816E+06
10.00	4.7954E+06	5.4254E+06	6.2213E+06

APPENDIX B. A Procedure for Selecting a Gamma Prior Distribution

It is required to find a and b which satisfies

$$\int_{LL}^{UL} \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx} dx = p_0, \tag{B1}$$

where

$$\int_{0}^{LL} \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx} dx = \int_{UL}^{\infty} \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx} dx = (1-p_0)/2.$$

Letting y = x/LL in (B1), we have

$$\int_{1}^{UL/LL} \frac{(bLL)^a}{\Gamma(a)} y^{a-1} e^{-(bLL)y} dy = p_0. \tag{B2}$$

Since b is a scale parameter, set bLL=1, and solve (B2) for the shape parameter a. Thus a depends only upon the value of UL/LL, or equivalently, log(UL/LL). Once a has been numerically determined, we can solve (B2) for a temporary value of b, say b_0, corresponding to a temporary lower limit of, say, $1.0x10^{-6}$f/h. Since b is a scale parameter, we know that

$$bLL = b_0(1.0x10^{-6})$$

from which

$$b = b_0(1.0x10^{-6} f/h)/LL.$$

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A Bayesian-Zero-Failure (BAZE) reliability demonstration testing procedure is presented. The method is developed for an exponential failure-time model and a gamma prior distribution on the failure-rate. A simple graphical approach using percentiles is used to fit the prior distribution. The procedure is given in an easily applied step-by-step form which does not require the use of a computer for its implementation.

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The BAZE approach is used to obtain sample test plans for selected components of nuclear reactor safety systems.

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